Math 1553 Worksheet: Chapter 5 and 6.1

1. Let $A = \begin{pmatrix} 2 & -8 & 6 & 8 \\ 3 & -9 & 5 & 10 \\ -3 & 0 & 1 & -2 \\ 1 & -4 & 0 & 6 \end{pmatrix}$.

a) Compute $\det(A)$ using row reduction.

b) Compute $\det(A^{-1})$ without doing any more work.

c) Compute $\det((A^T)^5)$ without doing any more work.

2. Play matrix tic-tac-toe!

Instead of X against O, we have 1 against 0. The 1-player wins if the final matrix has nonzero determinant, while the 0-player wins if the determinant is zero. You can change who goes first, and you can also modify the size of the matrix.

Click the link above, or copy and paste the url below:

`http://textbooks.math.gatech.edu/ila/demos/tictactoe/tictactoe.html`

Can you think of a winning strategy for the 0 player who goes first in the 2×2 case? Is there a winning strategy for the 1 player if they go first in the 2×2 case?

3. True or false. If the statement is always true, answer true and justify why it is true. Otherwise, answer false and justify why it is false. In every case, assume that A is an $n \times n$ matrix.

a) To find the eigenvectors of A, we reduce the matrix A to row echelon form.

b) If v_1 and v_2 are linearly independent eigenvectors of A, then they must correspond to different eigenvalues.

4. In what follows, T is a linear transformation with matrix A. Find the eigenvectors and eigenvalues of A without doing any matrix calculations. (Draw a picture!)

a) $T =$ projection onto the xz-plane in $\mathbf{R}^3$.

b) $T =$ reflection over $y = 2x$ in $\mathbf{R}^2$.