Name:________________________________ Recitation Section:____________

Math 1553 Quiz 4, Spring 2019 (10 points, 10 minutes)

Solutions

Show your work unless told otherwise, or you may receive little or no credit.

1. Answer each question. No justification is required and there is no partial credit.

a) The set of solutions to a homogeneous system of four linear equations in five unknowns is a subspace of $\mathbf{R}^4$.
FALSE (it is Nul A for a 4×5 matrix, so a subspace of $\mathbf{R}^5$)

b) Consider the subspace $V = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \mid x - 3y + 4z = 0 \right\}$ of $\mathbf{R}^3$, which contains the vectors $\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} -4 \\ 0 \\ 1 \end{pmatrix}$. Is $\left\{ \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -4 \\ 0 \\ 1 \end{pmatrix} \right\}$ a basis for V?
YES (two linearly independent vectors in a 2-dimensional subspace)

c) Consider the matrix $A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 0 & 0 \end{pmatrix}$, which has RREF $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 \end{pmatrix}$.

Is $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$ a basis for Col$A$? NO

2. (3 points) Write a matrix A so that its matrix transformation $T(x) = Ax$ has domain $\mathbf{R}^4$ and has range equal to the xy-plane within $\mathbf{R}^3$.

Solution: We need A to be a 3×4 matrix with the xy-plane as its column span. For example,

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{or} \quad A = \begin{pmatrix} 1 & -1 & 33 & 0 \\ 2 & 7 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \text{etc.}$$

Turn to the back side!

3. (1 point each) In each case, a matrix is given below.

Match each matrix to its corresponding transformation (choosing from (i) through (viii)) by writing that roman numeral next to the matrix. Note there are four matrices and eight options, so not every option is used.

$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ This is (i) Reflection across x-axis

$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ This is (v) Rotation counterclockwise by $\pi/2$ radians

$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ This is (viii) Projection onto the y-axis

$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ This is (iii) Scaling by a factor of 2

(i) Reflection across x-axis
(ii) Reflection across y-axis
(iii) Scaling by a factor of 2
(iv) Scaling by a factor of $1/2$
(v) Rotation counterclockwise by $\pi/2$ radians
(vi) Rotation clockwise by $\pi/2$ radians
(vii) Projection onto the x-axis
(viii) Projection onto the y-axis