Math 1553 Supplement §4.4 and 4.5

1. Find all matrices B that satisfy

$$\begin{pmatrix} 1 & -3 \\ -3 & 5 \end{pmatrix} B = \begin{pmatrix} -3 & -11 \\ 1 & 17 \end{pmatrix}.$$

2. Let T and U be the (linear) transformations below:

$T(x_1, x_2, x_3) = (x_3 - x_1,\ x_2 + 4x_3,\ x_1,\ 2x_2 + x_3)$ $\qquad U(x_1, x_2, x_3, x_4) = (x_1 - 2x_2,\ x_1)$.

a) Which compositions makes sense (circle all that apply)? $\quad U \circ T \qquad T \circ U$

b) Compute the standard matrix for T and for U.

c) Compute the standard matrix for each composition that you circled in (a).

3. True or false (justify your answer). Answer true if the statement is *always* true. Otherwise, answer false.

a) If A and B are matrices and the products AB and BA are both defined, then A and B must be square matrices with the same number of rows and columns.

b) If A, B, and C are nonzero 2×2 matrices satisfying $BA = CA$, then $B = C$.

c) Suppose A is an 4×3 matrix whose associated transformation $T(x) = Ax$ is not one-to-one. Then there must be a 3×3 matrix B which is not the zero matrix and satisfies $AB = 0$.

d) Suppose $T : \mathbf{R}^n \to \mathbf{R}^m$ and $U : \mathbf{R}^m \to \mathbf{R}^p$ are one-to-one linear transformations. Then $U \circ T$ is one-to-one. (What if U and T are not necessarily linear?)

4. **a)** Fill in: A and B are invertible $n \times n$ matrices, then the inverse of AB is ________.

b) If the columns of an $n \times n$ matrix Z are linearly independent, is Z necessarily invertible? Justify your answer.

c) If A and B are $n \times n$ matrices and $ABx = 0$ has a unique solution, does $Ax = 0$ necessarily have a unique solution? Justify your answer.

5. In each case, use geometric intuition to either give an example of a matrix with the desired properties or explain why no such matrix exists.

a) A 3×3 matrix P, which is not the identity matrix or the zero matrix, and satisfies $P^2 = P$.

b) A 2×2 matrix A satisfying $A^2 = I$.

c) A 2×2 matrix A satisfying $A^3 = -I$.