Math 1553 Worksheet §5.1, 5.2

1. True or false. If the statement is always true, answer true and justify why it is true. Otherwise, answer false and give an example that shows it is false. In every case, assume that A is an $n \times n$ matrix.

a) To find the eigenvectors of A, we reduce the matrix A to row echelon form.

b) If v_1 and v_2 are linearly independent eigenvectors of A, then they must correspond to different eigenvalues.

2. In what follows, T is a linear transformation with matrix A. Find the eigenvectors and eigenvalues of A without doing any matrix calculations. (Draw a picture!)

a) $T =$ projection onto the xz-plane in $\mathbf{R}^3$.

b) $T =$ reflection over $y = 2x$ in $\mathbf{R}^2$.

3. Consider the matrix

$$A = -\frac{1}{5}\begin{pmatrix} 8 & 3 \\ 2 & 7 \end{pmatrix}.$$

Find, draw, and label the eigenspaces of A.

To save time, you may use the fact that the characteristic polynomial of A is

$$\det(A - \lambda I) = (\lambda + 2)(\lambda + 1).$$