Name:____________________ Studio Section:__________

Math 1553 Quiz 4, Fall 2019 (10 points, 10 minutes)

Solutions

Show your work on problem 3 or you may receive little or no credit.

1. (1 point each) True or false. If the statement is *always* true, answer TRUE. Otherwise, circle FALSE.

a) The matrix transformation $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ performs reflection across the x-axis in $\mathbf{R}^2$. TRUE $\boxed{\text{FALSE}}$ (T reflects across the y-axis then projects onto the x-axis)

b) The matrix transformation $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ performs rotation counterclockwise by $90°$ in $\mathbf{R}^2$. TRUE $\boxed{\text{FALSE}}$ (T rotates clockwise $90°$)

2. (2 points) Fill in the blanks: If A is a 7×6 matrix and the solution set for $Ax = 0$ is a plane, then the column space of A is a __4__-dimensional subspace of $\mathbf{R}^{\boxed{7}}$.

Reason: $\text{rank}(A) + \text{nullity}(A) = 6$ $\text{rank}(A) + 2 = 6$ $\text{rank}(A) = 4$

3. (6 points) Let $A = \begin{pmatrix} 1 & 1 & 2 & 1 \\ -1 & 0 & -1 & -2 \\ 2 & 2 & 4 & 2 \end{pmatrix}$. Find a basis for Col$A$ and a basis for Nul$A$.

Solution: We row-reduce $\left(A \mid 0\right)$:

$$\left(\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 0 \\ -1 & 0 & -1 & -2 & 0 \\ 2 & 2 & 4 & 2 & 0 \end{array}\right) \xrightarrow[R_3=R_3-2R_1]{R_2=R_2+R_1} \left(\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{R_1=R_1-R_2} \left(\begin{array}{cccc|c} 1 & 0 & 1 & 2 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right).$$

We see x_3 and x_4 are free, and $x_1 = -x_3 - 2x_4$ and $x_2 = -x_3 + x_4$. The parametric vector form for elements of Nul A is:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -x_3 - 2x_4 \\ -x_3 + x_4 \\ x_3 \\ x_4 \end{pmatrix} = x_3\begin{pmatrix} -1 \\ -1 \\ 1 \\ 0 \end{pmatrix} + x_4\begin{pmatrix} -2 \\ 1 \\ 0 \\ 1 \end{pmatrix}.$$

A basis for Nul A is $\left\{\begin{pmatrix} -1 \\ -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 0 \\ 1 \end{pmatrix}\right\}$.

A basis for Col A is given by the pivot columns of A, namely $\left\{\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}\right\}$. In this case, any two columns of A will actually form a basis for Col$A$, so any two columns of A will be a correct answer.