## Supplemental problems: §3.2

**1.** Let $A$ be a $3\times 4$ matrix with column vectors $v_1, v_2, v_3, v_4$, and suppose $v_2 = 2v_1 - 3v_4$. Consider the matrix transformation $T(x) = Ax$.

**a)** Is it possible that $T$ is one-to-one? If yes, justify why. If no, find distinct vectors $v$ and $w$ so that $T(v) = T(w)$.

**b)** Is it possible that $T$ is onto? Justify your answer.

**2.** **a)** Which of the following are onto transformations? (Check all that apply.)

- ☐ $T : \mathbf{R}^3 \to \mathbf{R}^3$, reflection over the $xy$-plane
- ☐ $T : \mathbf{R}^3 \to \mathbf{R}^3$, projection onto the $xy$-plane
- ☐ $T : \mathbf{R}^3 \to \mathbf{R}^2$, project onto the $xy$-plane, forget the $z$-coordinate
- ☐ $T : \mathbf{R}^2 \to \mathbf{R}^2$, scale the $x$-direction by 2

**b)** Let $A$ be a square matrix and let $T(x) = Ax$. Which of the following guarantee that $T$ is onto? (Check all that apply.)

- ☐ $T$ is one-to-one
- ☐ $Ax = 0$ is consistent

**3.** Find all real numbers $h$ so that the transformation $T : \mathbf{R}^3 \to \mathbf{R}^2$ given by

$$T(v) = \begin{pmatrix} -1 & 0 & 2-h \\ h & 0 & 3 \end{pmatrix} v$$

is onto.

## Supplemental problems: §3.3

**1.** Circle **T** if the statement is always true, and circle **F** otherwise.

a) **T** **F** If $T : \mathbf{R}^n \to \mathbf{R}^n$ is linear and $T(e_1) = T(e_2)$, then the homogeneous equation $T(x) = 0$ has infinitely many solutions.

b) **T** **F** If $T : \mathbf{R}^n \to \mathbf{R}^\mathbf{m}$ is a one-to-one linear transformation and $m \neq n$, then $T$ must not be onto.

**2.** Consider $T : \mathbf{R}^3 \to \mathbf{R}^3$ given by

$$T(x, y, z) = (x,\ x + z,\ 3x - 4y + z,\ x).$$

Is $T$ one-to-one? Justify your answer.

**3.** Which of the following transformations $T$ are onto? Which are one-to-one? If the transformation is not onto, find a vector not in the range. If the matrix is not one-to-one, find two vectors with the same image.

**a)** The transformation $T : \mathbf{R}^3 \to \mathbf{R}^2$ defined by $T(x, y, z) = (y, y)$.

**b)** JUST FOR FUN: Consider $T$ : (Smooth functions) $\to$ (Smooth functions) given by $T(f) = f'$ (the derivative of $f$). Then $T$ is not a transformation from any $\mathbf{R}^n$ to $\mathbf{R}^m$, but it is still *linear* in the sense that for all smooth $f$ and $g$ and all scalars $c$ (by properties of differentiation we learned in Calculus 1):

$$T(f + g) = T(f) + T(g) \quad \text{since} \quad (f + g)' = f' + g'$$
$$T(cf) = cT(f) \quad \text{since} \quad (cf)' = cf'.$$

Is $T$ one-to-one?

**4.** In each case, determine whether $T$ is linear. Briefly justify.

**a)** $T(x_1, x_2) = (x_1 - x_2, x_1 + x_2, 1)$.

**b)** $T(x, y) = (y, x^{1/3})$.

**c)** $T(x, y, z) = 2x - 5z$.

**5.** The second little pig has decided to build his house out of sticks. His house is shaped like a pyramid with a triangular base that has vertices at the points $(0, 0, 0)$, $(2, 0, 0)$, $(0, 2, 0)$, and $(1, 1, 1)$.

The big bad wolf finds the pig's house and blows it down so that the house is rotated by an angle of45° in a counterclockwise direction about the $z$-axis (look downward onto the $xy$-plane the way we usually picture the plane as $\mathbf{R}^2$), and then projected onto the $xy$-plane.

In the worksheet, we found the matrix for the transformation $T$ caused by the wolf. Geometrically describe the image of the house under $T$.