Name:______________________________ Recitation Section:__________

Math 1553 Quiz 7: 5.3 (10 points, 10 minutes)

Solutions

Show your work and justify answers where appropriate. If you write the correct answer without sufficient work or justification, you will receive little or no credit.

1. True or false, 1 point each. If the statement is *always* true, answer true. Otherwise, answer false. You do not need to justify your answer.

a) If A is a 3×3 matrix and its eigenvalues are -1 and 4, then A is not diagonalizable. TRUE **[FALSE]**

b) The matrices $A = \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}$ are similar.

TRUE **[FALSE]**

c) If A is an $n \times n$ diagonal matrix, then A is diagonalizable. **[TRUE]** FALSE

Solution.

a) False. For example, $A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 4 \end{pmatrix}$.

b) False: A and B don't even have the same eigenvalues.

c) True. $A = IAI^{-1}$.

2. (2 points) Write a 2×2 matrix A which is not diagonalizable. You do not need to justify your answer.

Solution.

Many examples, such as

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix}.$$

3. (5 points) Write the matrix A whose eigenvectors are $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and whose eigenvalues (in the same order) are -1 and 2.

Solution.

$A = PDP^{-1}$ where $P = \begin{pmatrix} 1 & 0 \\ -3 & 1 \end{pmatrix}$ and $D = \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}$. We compute $P^{-1} = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$.

$$A = \begin{pmatrix} 1 & 0 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 6 & 2 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 9 & 2 \end{pmatrix}.$$