Name:____________________ Recitation Section:__________

Math 1553 Quiz 6: 5.1, 5.2 (10 points, 10 minutes)

Solutions

Show your work and justify answers where appropriate. If you write the correct answer without sufficient work or justification, you will receive little or no credit.

1. True or false, 1 point each. If the statement is *always* true, answer true. Otherwise, answer false.

a) If we row-reduce an $n \times n$ matrix A to obtain a matrix B, then A and B have the same eigenvalues. TRUE $\boxed{\text{FALSE}}$

b) Suppose A is an $n \times n$ matrix. Then 3 is an eigenvalue of A if and only if $\mathrm{Col}(A-3I)$ is not $\mathbf{R}^n$. $\boxed{\text{TRUE}}$ FALSE

c) The vector $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is an eigenvector of the matrix $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$. TRUE $\boxed{\text{FALSE}}$

Solution.

a) False. For example, $A = \begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix}$ has eigenvalues $\lambda = 2$ and $\lambda = 4$, but it can be quickly row-reduced to the identity matrix $B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ whose only eigenvalue is $\lambda = 1$.

b) True: 3 is an eigenvalue of A if and only if $A-3I$ is not invertible, if and only if the columns of $A-3I$ do not span $\mathbf{R}^n$.

c) False. The zero vector is never an eigenvector of any matrix.

2. (3 points) Find the eigenvalues of

$$A = \begin{pmatrix} -2 & 1 \\ 1 & -1 \end{pmatrix}.$$

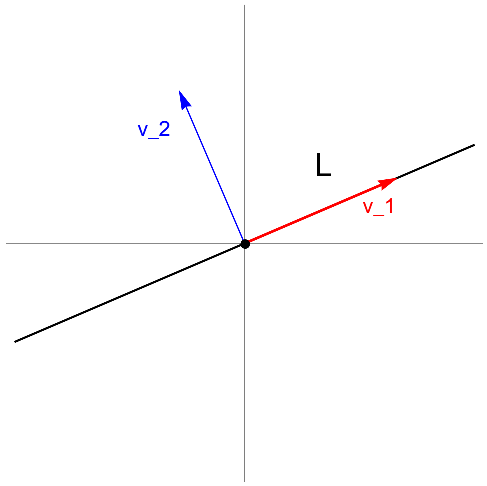
Solution.

$$0 = \det(A - \lambda I) = \det\begin{pmatrix} -2-\lambda & 1 \\ 1 & -1-\lambda \end{pmatrix} = (-2-\lambda)(-1-\lambda) - 1$$

$$= \lambda^2 + 3\lambda + 2 - 1 = \lambda^2 + 3\lambda + 1, \qquad \lambda = \frac{-3 \pm \sqrt{9-4}}{2} \qquad \boxed{\lambda = \frac{-3 \pm \sqrt{5}}{2}}.$$

Turn over to the back side for problem 3.

3. (4 points) Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation which reflects across the line L drawn below, and let A be the standard matrix for T.



a) Write all eigenvalues of A.

This problem is similar to an example in our class notes near the end of section 5.1, and its appearance on the quiz was inspired by #2(c) from the 5.1-5.2 worksheet, which is nearly the same problem.

$\lambda_1 = 1$ and $\lambda_2 = -1$.

(The equation of the line was not given, and it is irrelevant: A fixes every vector along the line L, while A flips every vector perpendicular to L.)

b) For each eigenvalue of A, draw one eigenvector on the graph above. Your eigenvector does not need to be perfect, but it should be reasonably accurate.

Above, v_1 corresponds to $\lambda_1 = 1$, while v_2 corresponds to $\lambda_2 = -1$.
Many answers are possible: v_1 can be any nonzero vector on L (going up-to-right or down-to-left), while v_2 can be any nonzero vector perpendicular to L (going up-to-left or down-to-right).

Algebraically, the problem would have been much more painful!
The line is actually the line $y = \frac{3}{7}x$, the matrix is $A = \frac{1}{29}\begin{pmatrix} 20 & 21 \\ 21 & -20 \end{pmatrix}$, and the eigenvectors drawn are $v_1 = \begin{pmatrix} 7/3 \\ 1 \end{pmatrix}$ and $v_2 = \begin{pmatrix} -1 \\ 7/3 \end{pmatrix}$.