Name:____________________________ Recitation Section:__________

Math 1553 Lecture A, Quiz 3: Section 1.3 (10 points, 10 minutes)

Solutions

1. (2 points) Write the following system of linear equations in x_1, x_2, and x_3 as a *vector equation*:

$$x_1 + x_3 = 4$$
$$3x_1 + 2x_2 - x_3 = 1$$

(you do not need to solve the vector equation).

Solution.

We separate and then pull out the constants:

$$x_1\begin{pmatrix}1\\3\end{pmatrix} + x_2\begin{pmatrix}0\\2\end{pmatrix} + x_3\begin{pmatrix}1\\-1\end{pmatrix} = \begin{pmatrix}4\\1\end{pmatrix}.$$

2. (1 point each) True or False. Circle TRUE if the statement is *always* true. Otherwise, circle FALSE.

a) If v_1 and v_2 are nonzero vectors in $\mathbf{R}^2$, then $\mathrm{Span}\{v_1, v_2\} = \mathbf{R}^2$.

TRUE $\boxed{\text{FALSE}}$

If v_1 and v_2 are on the same line, then $\mathrm{Span}\{v_1, v_2\}$ is just a line in $\mathbf{R}^2$.
We did an example in class, and there is one in the 1.3 PDF (page 10).

b) Asking whether the linear system corresponding to an augmented matrix $\left(\begin{array}{ccc|c} a_1 & a_2 & a_3 & b\end{array}\right)$ is consistent, amounts to asking whether b is in $\mathrm{Span}\{a_1, a_2, a_3\}$.

$\boxed{\text{TRUE}}$ FALSE

I took this from the 1.3 Webwork.

(Please turn over to the back side!)

On problem #3, show your work clearly. If you write the correct answer without appropriate work, you will receive little or no credit.

3. (6 points) Let

$$v_1 = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \qquad v_2 = \begin{pmatrix} 3 \\ 11 \\ 2 \end{pmatrix}, \qquad w = \begin{pmatrix} 5 \\ h \\ 8 \end{pmatrix}.$$

For what value (or values) of h is w is a linear combination of v_1 and v_2?

Solution.

From section 1.3 we know that w is a linear combination of v_1 and v_2 if and only if the system represented by the augmented matrix below is consistent.

$$\left(\begin{array}{cc|c} 1 & 3 & 5 \\ -2 & 11 & h \\ 3 & 2 & 8 \end{array}\right) \xrightarrow[R_3=R_3-3R_1]{R_2=R_2+2R_1} \left(\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & 17 & h+10 \\ 0 & -7 & -7 \end{array}\right) \xrightarrow{R_2 \leftrightarrow R_3} \left(\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & -7 & -7 \\ 0 & 17 & h+10 \end{array}\right) \xrightarrow{R_2=R_2 \cdot \frac{-1}{7}} \left(\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & 1 & 1 \\ 0 & 17 & h+10 \end{array}\right)$$

$$\xrightarrow{R_3=R_3-17R_2} \left(\begin{array}{cc|c} \boxed{1} & 3 & 5 \\ 0 & \boxed{1} & 1 \\ 0 & 0 & h-7 \end{array}\right).$$

The system is consistent precisely when the right column is *not* a pivot column, so we must have $h-7=0$.

$$\boxed{h=7}.$$

****If we had wanted to go a step further and write w as a linear combination of v_1 and v_2, we would put the matrix in RREF to solve $x_1v_1 + x_2v_2 = w$.

$$\left(\begin{array}{cc|c} \boxed{1} & 3 & 5 \\ 0 & \boxed{1} & 1 \\ 0 & 0 & 0 \end{array}\right) \xrightarrow{R_1=R_1-3R_2} \left(\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right), \qquad x_1 = 2, \quad x_2 = 1.$$

So $2v_1 + v_2 = w$.