Name:______________________________ Recitation Section:__________

Math 1553 Lecture C, Quiz 3: Section 1.3 (10 points, 10 minutes)

Solutions

1. (2 points) Write the following system of linear equations in x_1, x_2, and x_3 as a *vector equation*:

$$x_1 + x_3 = 4$$
$$3x_1 + 2x_2 - x_3 = 1$$

(you do not need to solve the vector equation).

Solution.

We separate and then pull out the constants:

$$x_1 \begin{pmatrix} 1 \\ 3 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ 2 \end{pmatrix} + x_3 \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}.$$

2. (1 point each) True or False. Circle TRUE if the statement is *always* true. Otherwise, circle FALSE.

a) Asking whether the linear system corresponding to an augmented matrix $\left(a_1 \quad a_2 \quad a_3 \mid b \right)$ is consistent, amounts to asking whether b is in Span$\{a_1, a_2, a_3\}$.

[TRUE] FALSE

I took this from the 1.3 Webwork.

b) If v_1 and v_2 are nonzero vectors in $\mathbf{R}^2$, then Span$\{v_1, v_2\} = \mathbf{R}^2$.

TRUE [FALSE]

If v_1 and v_2 are on the same line, then Span$\{v_1, v_2\}$ is just a line in $\mathbf{R}^2$. We did an example in class, and there is one in the 1.3 PDF (page 10).

(Please turn over to the back side!)

On problem #3, show your work clearly. If you write the correct answer without appropriate work, you will receive little or no credit.

3. (6 points) Let

$$v_1 = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \qquad v_2 = \begin{pmatrix} 3 \\ 11 \\ 2 \end{pmatrix}, \qquad w = \begin{pmatrix} 1 \\ h \\ -4 \end{pmatrix}.$$

For what value (or values) of h is w is a linear combination of v_1 and v_2?

Solution.

From section 1.3 we know that w is a linear combination of v_1 and v_2 if and only if the system represented by the augmented matrix below is consistent.

$$\left(\begin{array}{cc|c} 1 & 3 & 1 \\ -2 & 11 & h \\ 3 & 2 & -4 \end{array}\right) \xrightarrow[R_3=R_3-3R_1]{R_2=R_2+2R_1} \left(\begin{array}{cc|c} 1 & 3 & 1 \\ 0 & 17 & h+2 \\ 0 & -7 & -7 \end{array}\right) \xrightarrow{R_2 \longleftrightarrow R_3} \left(\begin{array}{cc|c} 1 & 3 & 1 \\ 0 & -7 & -7 \\ 0 & 17 & h+2 \end{array}\right) \xrightarrow{R_2=R_2\cdot\frac{-1}{7}} \left(\begin{array}{cc|c} 1 & 3 & 1 \\ 0 & 1 & 1 \\ 0 & 17 & h+2 \end{array}\right)$$

$$\xrightarrow{R_3=R_3-17R_2} \left(\begin{array}{cc|c} \boxed{1} & 3 & 1 \\ 0 & \boxed{1} & 1 \\ 0 & 0 & h-15 \end{array}\right).$$

The system is consistent precisely when the right column is *not* a pivot column, so we must have $h - 15 = 0$.

$$\boxed{h = 15}.$$

****If we had wanted to go a step further and write w as a linear combination of v_1 and v_2, we would put the matrix in RREF to solve $x_1v_1 + x_2v_2 = w$.

$$\left(\begin{array}{cc|c} \boxed{1} & 3 & 1 \\ 0 & \boxed{1} & 1 \\ 0 & 0 & 0 \end{array}\right) \xrightarrow{R_1=R_1-3R_2} \left(\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right), \qquad x_1 = -2, \quad x_2 = 1.$$

So $-2v_1 + v_2 = w$.