

Math 1553

Introduction to Linear Algebra

School of Mathematics
Georgia Institute of Technology

Introduction to Linear Algebra

Motivation and Overview

What is Linear Algebra?

Linear

- ▶ having to do with lines/planes/etc.
- ▶ For example, $x + y + 3z = 7$, not \sin , \log , x^2 , etc.

Algebra

- ▶ solving equations involving numbers and symbols
- ▶ from al-jabr (Arabic), meaning reunion of broken parts
- ▶ 9th century Abu Ja'far Muhammad ibn Muso al-Khwarizmi

Why a whole course?

But these are the easiest kind of equations! I learned how to solve them in 7th grade!

Ah, but engineers need to solve *lots* of equations in *lots* of variables.

$$\begin{aligned}3x_1 + 4x_2 + 10x_3 + 19x_4 - 2x_5 - 3x_6 &= 141 \\7x_1 + 2x_2 - 13x_3 - 7x_4 + 21x_5 + 8x_6 &= 2567 \\-x_1 + 9x_2 + \frac{3}{2}x_3 + x_4 + 14x_5 + 27x_6 &= 26 \\\frac{1}{2}x_1 + 4x_2 + 10x_3 + 11x_4 + 2x_5 + x_6 &= -15\end{aligned}$$

Often, it's enough to know some information about the set of solutions without having to solve the equations at all!

Also, what if one of the coefficients of the x_i is itself a parameter— like an unknown real number t ?

In real life, the difficult part is often in recognizing that a problem can be solved using linear algebra in the first place: need *conceptual* understanding.

Large classes of engineering problems, no matter how huge, can be reduced to linear algebra:

$$Ax = b \quad \text{or}$$

$$Ax = \lambda x$$

“... and now it's just linear algebra”

Civil Engineering: How much traffic flows through the four labeled segments?

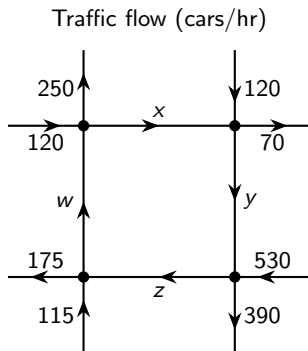
~~~~~> system of linear equations:

$$w + 120 = x + 250$$

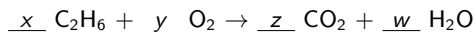
$$x + 120 = y + 70$$

$$y + 530 = z + 390$$

$$z + 115 = w + 175$$



**Chemistry:** Balancing reaction equations



~~~~~> system of linear equations, one equation for each element.

$$2x = z$$

$$6x = 2w$$

$$2y = 2z + w$$

Biology: In a population of rabbits. . .

- ▶ half of the new born rabbits survive their first year
- ▶ of those, half survive their second year
- ▶ the maximum life span is three years
- ▶ rabbits produce 0, 6, 8 rabbits in their first, second, and third years

If I know the population in 2016 (in terms of the number of first, second, and third year rabbits), then what is the population in 2017?

~~~~~> system of linear equations:

$$\begin{array}{rcl} & 6y_{2016} + 8z_{2016} & = x_{2017} \\ \frac{1}{2}x_{2016} & & = y_{2017} \\ & \frac{1}{2}y_{2016} & = z_{2017} \end{array}$$

## Question

Does the rabbit population have an asymptotic behavior? Is this even a linear algebra question? Yes, it is!



**Geometry and Astronomy:** Find the equation of a circle passing through 3 given points, say  $(1, 0)$ ,  $(0, 1)$ , and  $(1, 1)$ . The general form of a circle is  $a(x^2 + y^2) + bx + cy + d = 0$ .

~~~~~> system of linear equations:

$$\begin{aligned}a + b + d &= 0 \\a + c + d &= 0 \\2a + b + c + d &= 0\end{aligned}$$

Very similar to: compute the orbit of a planet:

$$ax^2 + by^2 + cxy + dx + ey + f = 0$$

Applications of Linear Algebra

Google: “The 25 billion dollar eigenvector.” Each web page has some importance, which it shares via outgoing links to other pages
~~~~~> system of linear equations (in gazillions of variables).

Larry Page flies around in a private 747 partly because he paid attention in his linear algebra class.

Observables like position and momentum  $\rightsquigarrow$  linear operators

The “wave function”  $\psi_t$  (where  $t$  is the time parameter), in some number of variables: satisfies Schrodinger’s equation for the system’s Hamiltonian.

How to do computations:

The value of an observable  $A$  at time  $t$  is the inner product (like a dot product) of  $\psi_t$  with  $A\psi_t$ . This is actually given by an integral!

In this way, quantum mechanics relates to linear algebra.

Unfortunately: this part is beyond the scope of Math 1553.

## Overview of the Course

- ▶ Solve the matrix equation  $Ax = b$ 
  - ▶ **Solve systems of linear equations** using matrices, row reduction, and inverses.
  - ▶ **Solve systems of linear equations with varying parameters** using parametric forms for solutions, the geometry of linear transformations, the characterizations of invertible matrices, and determinants.
  
- ▶ Solve the matrix equation  $Ax = \lambda x$ 
  - ▶ **Solve eigenvalue problems** through the use of the characteristic polynomial.
  - ▶ **Understand the dynamics of a linear transformation** via the computation of eigenvalues, eigenvectors, and diagonalization.
  
- ▶ Almost solve the equation  $Ax = b$ 
  - ▶ **Find best-fit solutions to systems of linear equations that have no actual solution** using least squares approximations.

## What to Expect This Semester

Your previous math courses probably focused on how to do (sometimes rather involved) computations.

- ▶ Compute the derivative of  $\sin(\log x) \cos(e^x)$ .
- ▶ Compute  $\int_0^1 (1 - \cos(x)) dx$ .

This is important, **but** Wolfram Alpha can do all these problems better than any of us can. Nobody is going to hire us to do something a computer can do better.

If a computer can do the problem better than we can, then it's just an algorithm: this is not real problem solving.

So what are we going to do?

- ▶ About half the material focuses on how to do linear algebra computations—that is still important.
- ▶ The other half is on *conceptual* understanding of linear algebra. This is much more subtle: it's about figuring out *what question* to ask the computer, or whether you actually need to do any computations at all.

See our [lecture's webpage](#) for the following.

- ▶ **Calendar:** lists what will happen on which day, links to daily slides, quizzes, practice exams, solutions, etc.
- ▶ A syllabus which provides information such as:
  - ▶ **Course administration:** the names of your TAs, their office hours, your recitation location, etc., are on the syllabus.
  - ▶ **Course organization:** grading policies, details about homework and exams, etc.
- ▶ A link to the master website for Math 1553, which includes advice and help: how to succeed in this course, helpful links, and more.

**Canvas:** your grades, link to WeBWork.

**Piazza:** forum for asking questions and discussing mathematics. We will be using Piazza for the class participation grade, starting the second week of class.