

**MATH 1553, SPRING 2018**  
**SAMPLE MIDTERM 2 (VERSION A), 1.7 THROUGH 2.9**

<b>Name</b>		<b>Section</b>	
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Please **read all instructions** carefully before beginning.

- You have 50 minutes to complete this exam.
- There are no aids of any kind (calculators, notes, text, etc.) allowed.
- Please show your work unless specified otherwise. A correct answer without appropriate work may be given little or no credit.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

This is a practice exam. It is similar in format, length, and difficulty to the real exam. It is **not** meant as a comprehensive list of study problems. I recommend completing the practice exam in 50 minutes, without notes or distractions.

The exam is not designed to test material from the previous midterm on its own. However, knowledge of the material prior to section §1.7 is necessary for everything we do for the rest of the semester, so it is fair game for the exam as it applies to §§1.7 through 2.9.

## Problem 1.

[Parts a) through f) are worth 2 points each]

- a) Complete the following definition (be mathematically precise!):  
A set of vectors  $\{v_1, v_2, \dots, v_p\}$  in  $\mathbf{R}^n$  is *linearly independent* if...

- b) Let  $A = \begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix}$ . If  $A$  is invertible, find  $A^{-1}$ . If  $A$  is not invertible, justify why.

The remaining problems are true or false. Answer true if the statement is *always* true. Otherwise, answer false. You do not need to justify your answer.

- c) **T** **F** If  $A$  is an  $n \times n$  matrix and the columns of  $A$  span  $\mathbf{R}^n$ , then  $Ax = 0$  has only the trivial solution.
- d) **T** **F** If  $A$  is a  $6 \times 7$  matrix and the null space of  $A$  has dimension 4, then the column space of  $A$  is a 2-plane.
- e) **T** **F** If  $A$  is an  $n \times n$  matrix and  $Ax = b$  has exactly one solution for some  $b$  in  $\mathbf{R}^n$ , then  $A$  is invertible.
- f) **T** **F** If  $A$  is an  $m \times n$  matrix and  $m > n$ , then the linear transformation  $T(x) = Ax$  cannot be one-to-one.

## Problem 2.

[10 points]

Parts (a), (b), and (c) are unrelated.

a) Let  $V = \text{Span} \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$ .

Fill in the blank: the dimension of  $V$  is \_\_\_\_\_.

b) Let  $W = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ in } \mathbf{R}^3 \mid x - y - z = 0 \right\}$ .

Is  $W$  a subspace of  $\mathbf{R}^3$ ? (no justification required)

YES

NO

c) The famous philologist is obsessed with the set of vectors

$$\left\{ \begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ h \\ -7 \end{pmatrix} \right\}$$

where  $h$  is some real number.

Find all values of  $h$  that make the set linearly dependent.

### Problem 3.

[11 points]

Let  $T : \mathbf{R}^3 \rightarrow \mathbf{R}^2$  be the linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_3 - x_2 \\ 2x_1 \end{pmatrix},$$

and let  $U : \mathbf{R}^2 \rightarrow \mathbf{R}^2$  be reflection about the line  $y = x$ .

- a) Write the standard matrix  $A$  for  $T$  and the standard matrix  $B$  for  $U$ .
- b) Is  $U$  one-to-one? Briefly justify your answer.
- c) Find the standard matrix for  $U \circ T$ .
- d) Is the transformation  $U \circ T$  onto? Briefly justify your answer.

## Problem 4.

[10 points]

Consider the following matrix  $A$  and its reduced row echelon form:

$$\begin{pmatrix} 1 & -2 & 4 \\ 0 & 0 & 1 \\ 1 & -2 & 3 \\ -2 & 4 & -8 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & -2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

- a) Find a basis for  $\text{Nul } A$ .
- b) Find a basis  $\mathcal{B}$  for  $\text{Col } A$ .

c) Let  $x = \begin{pmatrix} -2 \\ -1 \\ -1 \\ 4 \end{pmatrix}$ . Is  $x$  in  $\text{Col } A$ ?

If your answer is no, justify why  $x$  is not in  $\text{Col } A$ .

If your answer is yes, find  $[x]_{\mathcal{B}}$ .

## Problem 5.

[7 points]

Parts (a) and (b) are unrelated.

a) Suppose that a linear transformation  $T : \mathbf{R}^2 \rightarrow \mathbf{R}^2$  satisfies  $T \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$  and

$$T \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}. \text{ Find } T \begin{pmatrix} 4 \\ -1 \end{pmatrix}.$$

b) Write a single matrix  $A$  that satisfies both of the following two properties:

- Col  $A$  is a subspace of  $\mathbf{R}^4$ , and
- Nul  $A$  is the line  $y = 10x$  in  $\mathbf{R}^2$ .

[Scratch work]