Math 1553 Worksheet §5.1, 5.2

1. True or false. If the statement is always true, answer true and justify why it is true. Otherwise, answer false and give an example that shows it is false. In every case, assume that A is an $n \times n$ matrix.

a) The entries on the main diagonal of A are the eigenvalues of A.

b) The number λ is an eigenvalue of A if and only if there is a nonzero solution to the equation $(A - \lambda I)x = 0$.

c) To find the eigenvectors of A, we reduce the matrix A to row echelon form.

d) If v_1 and v_2 are linearly independent eigenvectors of A, then they must correspond to different eigenvalues.

e) If A is invertible and 2 is an eigenvalue of A, then $\frac{1}{2}$ is an eigenvalue of A^{-1}.

2. In what follows, T is a linear transformation with matrix A. Find the eigenvectors and eigenvalues of A without doing any matrix calculations. (Draw a picture!)

a) $T =$ identity transformation of $\mathbf{R}^3$.

b) $T =$ projection onto the xz-plane in $\mathbf{R}^3$.

c) $T =$ reflection over $y = 2x$ in $\mathbf{R}^2$.

3. Let $A = \begin{pmatrix} 5 & -2 & 3 \\ 0 & 1 & 0 \\ 6 & 7 & -2 \end{pmatrix}$. Find the eigenvalues of A.