Math 1553 Worksheet §2.1, 2.2, 2.3

1. If A is a 3×5 matrix and B is a 3×2 matrix, which of the following are defined?

a) $A - B$

b) AB

c) $A^T B$

d) A^2

2. True or false (justify your answer). Answer true if the statement is *always* true. Otherwise, answer false.

a) If A is an $n \times n$ matrix and the equation $Ax = b$ has at least one solution for each b in $\mathbf{R}^n$, then the solution is *unique* for each b in $\mathbf{R}^n$.

b) If A is an $n \times n$ matrix and every vector in $\mathbf{R}^n$ can be written as a linear combination of the columns of A, then A is invertible.

c) If A and B are invertible $n \times n$ matrices, then $A + B$ is invertible and

$$(A + B)^{-1} = A^{-1} + B^{-1}.$$

d) If A is an $m \times n$ matrix and B is an $n \times p$ matrix, then each column of AB is a linear combination of the columns of A.

e) If $AB = BC$ and B is invertible, then $A = C$.

3. Suppose A is an invertible 3×3 matrix and

$$A^{-1}e_1 = \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix}, \quad A^{-1}e_2 = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}, \quad A^{-1}e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

Find A.