**Math 1553 Supplement §1.7, 1.8, 1.9**

**1.** Justify why each of the following true statements can be checked without row reduction.

**a)** $\left\{ \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ \pi \end{pmatrix}, \begin{pmatrix} 0 \\ \sqrt{2} \\ 0 \end{pmatrix} \right\}$ is linearly independent.

**b)** $\left\{ \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ 10 \\ 20 \end{pmatrix}, \begin{pmatrix} 0 \\ 5 \\ 7 \end{pmatrix} \right\}$ is linearly independent.

**c)** $\left\{ \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ 10 \\ 20 \end{pmatrix}, \begin{pmatrix} 0 \\ 5 \\ 7 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$ is linearly dependent.

**2.** Let $A$ be a $3\times 4$ matrix with column vectors $v_1, v_2, v_3, v_4$. Suppose that $v_2 = 2v_1 - 3v_4$. Find one non-trivial solution to the equation $Ax = 0$.

**3.** Which of the following transformations $T$ are onto? Which are one-to-one? If the transformation is not onto, find a vector not in the range. If the matrix is not one-to-one, find two vectors with the same image.

**a)** The transformation $T : \mathbf{R}^3 \to \mathbf{R}^2$ defined by $T(x, y, z) = (0, x)$.

**b)** JUST FOR FUN: Consider $T$ : (Smooth functions) $\to$ (Smooth functions) given by $T(f) = f'$ (the derivative of $f$). Then $T$ is not a transformation from any $\mathbf{R}^n$ to $\mathbf{R}^m$, but it is still *linear* in the sense that for all smooth $f$ and $g$ and all scalars $c$ (by properties of differentiation we learned in Calculus 1):

$$T(f+g) = T(f) + T(g) \qquad ((f+g)' = f' + g')$$
$$T(cf) = cT(f) \qquad (cf)' = cf'.$$

Is $T$ one-to-one?

**4.** For each matrix $A$, describe what the associated matrix transformation $T$ does to $\mathbf{R}^3$ geometrically.

**a)** $\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ **b)** $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.