Name:______________________________ Recitation Section:__________

Math 1553 Quiz 6, Fall 2018: Sections 5.1-5.3 (10 points, 10 minutes)

Solutions

Show your work and justify answers unless instructed otherwise, or you will receive little or no credit.

1. (1 point) Suppose A and B are $n \times n$ matrices and that AB is invertible. Must it be true that A and B are both invertible? YES NO

Solution.

Yes. $\det(AB) = \det(A)\det(B) \neq 0$, therefore $\det(A) \neq 0$ and $\det(B) \neq 0$, so both A and B are invertible.

2. (6 pts) Suppose $\det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = 3$.

a) Find $\det\left(2\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}\right)$.

Let's call the matrix given in the problem M. Since M is a 3×3 matrix,

$$\det(2M) = 2^3\det(M) = 2^3(3) = 24.$$

b) Find $\det(A^{-1})$ if $A = \begin{pmatrix} -2a+d & -2b+e & -2c+f \\ a & b & c \\ g & h & i \end{pmatrix}$.

To get A, we start with the matrix given in the problem, swap the first two rows, then do a row replacement (adding $-2R_2$ to R_1). All of this just multiplies the original matrix's determinant by -1. Therefore,

$$\det(A) = -3, \quad \text{so} \quad \det(A^{-1}) = -\frac{1}{3}.$$

Turn over for problem #3

3. (3 points) Find all values of k (if there are any) so that

$$\det\begin{pmatrix} 1 & k & 3 \\ 0 & 0 & k \\ k & 1 & 2 \end{pmatrix} = 0.$$

We solve using the cofactor expansion of the determinant along the second row:

$$0 = k(-1)^{2+3}\det\begin{pmatrix} 1 & k \\ k & 1 \end{pmatrix} = -k(1-k^2), \qquad \text{so} \qquad 0 = -k(1-k)(1+k).$$

Our values of k are $\boxed{k=0,\ k=1,\ k=-1}$.