Name:______________________________ Recitation Section:__________

Math 1553 Quiz 4, Fall 2018: Sections 3.7, 3.9, 4.1 (10 points, 10 minutes)

Solutions

You do not need to show work on this quiz, except in #3d.

1. Fill in the blank: If A is a 4×7 matrix and the RREF of A has exactly two rows of zeros, then

$$\dim(\text{Col } A) = \underline{\ 2\ } \qquad \text{and} \qquad \dim(\text{Nul } A) = \underline{\ 5\ }.$$

The Rank Theorem states that $\dim(\text{Col } A) + \dim(\text{Nul } A) = 7$. From the RREF of A we know that A will have $4 - 2 = 2$ pivots, so $\dim(\text{Col } A) = 2$ and thus $\dim(\text{Nul} A) = 5$.

2. (2 points) True or false. Circle TRUE if the statement is always true. Otherwise, circle FALSE. You do not need to justify your answer.

a) If A is a 3×5 matrix, then the pivot columns of A form a basis for $\mathbf{R}^3$.

TRUE **[FALSE]** (might not have three linearly independent columns)

b) Suppose V is a subspace satisfying $\dim(V) = 3$ and that v_1, v_2, v_3 is a linearly independent set of vectors in V. Then $\{v_1, v_2, v_3\}$ must be a basis for V.

[TRUE] FALSE

True by the Basis Theorem.

Turn over to the back page!

3. Consider the following matrix A and its reduced row echelon form:

$$A = \begin{pmatrix} -2 & 10 & -5 & -2 \\ 1 & -5 & 0 & 1 \\ -1 & 5 & -6 & -1 \end{pmatrix} \xrightarrow{RREF} \begin{pmatrix} 1 & -5 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Define a matrix transformation by $T(x) = Ax$.

a) (1 point) What is the domain of T? (no justification required)
$\mathbf{R}^4$

b) (1 point) What is the codomain of T? (no justification required)
$\mathbf{R}^3$

c) (2 points) Write a basis for the range of T. (no justification required)
Since range(T)= Col(A) we just use the pivot columns of A:

$$\left\{ \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -5 \\ 0 \\ -6 \end{pmatrix} \right\}.$$

d) (2 point) Find one nonzero vector in Nul$A$.
We've been given the RREF of $(A \mid 0)$, which yields $x_1 - 5x_2 + x_4 = 0$ and $x_3 = 0$, where x_2 and x_4 are free. Any nonzero vector satisfying this is fine. The parametric form of the general solution would be

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = x_2 \begin{pmatrix} 5 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix},$$

so the most natural candidates are $\begin{pmatrix} 5 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$.