Math 1553, Extra Practice for Midterm 1 (through §3.4)

1. In this problem, A is an $m \times n$ matrix (m rows and n columns) and b is a vector in $\mathbf{R}^m$. Circle **T** if the statement is always true (for any choices of A and b) and circle **F** otherwise. Do not assume anything else about A or b except what is stated.

a) **T** **F** The matrix below is in reduced row echelon form.

$$\left(\begin{array}{cccc|c} 1 & 1 & 0 & -3 & 1 \\ 0 & 0 & 1 & -1 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

b) **T** **F** If A has fewer than n pivots, then $Ax = b$ has infinitely many solutions.

c) **T** **F** If the columns of A span $\mathbf{R}^m$, then $Ax = b$ must be consistent.

d) **T** **F** If $Ax = b$ is consistent, then the solution set is a span.

2. **a)** Is $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ in the span of $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$? Justify your answer.

b) What best describes Span $\left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$? Justify your answer.

(I) It is a plane through the origin.

(II) It is three lines through the origin.

(III) It is all of $\mathbf{R}^3$.

(IV) It is a plane, plus the line through the origin and the vector $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$.

c) Does Span $\left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \\ 3 \end{pmatrix} \right\} = \mathbf{R}^3$? If yes, justify your answer. If not, write a vector in $\mathbf{R}^3$ which is not in Span $\left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \\ 3 \end{pmatrix} \right\}$.

3. Let $v_1 = \begin{pmatrix} 1 \\ k \end{pmatrix}$, $v_2 = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$, and $b = \begin{pmatrix} 1 \\ h \end{pmatrix}$.

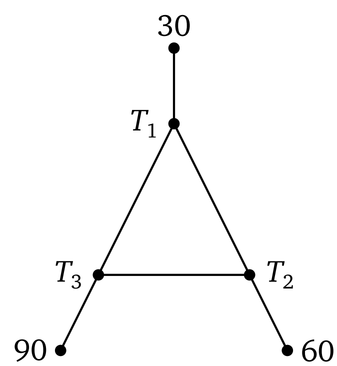
a) Find all values of h and k so that $x_1v_1 + x_2v_2 = b$ has infinitely many solutions.

b) Find all values of h and k so that b is *not* in $\mathrm{Span}\{v_1, v_2\}$.

c) Find all values of h and k so that there is exactly one way to express b as a linear combination of v_1 and v_2.

4. Let $A = \begin{pmatrix} 5 & -5 & 10 \\ 3 & -3 & 6 \end{pmatrix}$. Draw the column span of A.

5. The diagram below represents the temperature at points along wires, in celcius.



Let T_1, T_2, T_3 be the temperatures at the interior points. Assume the temperature at each interior point is the average of the temperatures of the three adjacent points.

a) Write a system of three linear equations whose solution would give the temperatures T_1, T_2, and T_3. Do not solve it.

b) Write the system as a vector equation. Do not solve it.

c) Write a matrix equation $Ax = b$ that represents this system. Specify every entry of A, x, and b. Do not solve it.

6. For each of the following, give an example if it is possible. If it is not possible, justify why there is no such example.

a) A 3×4 matrix A in RREF with 2 pivot columns, so that for some vector b, the system $Ax = b$ has exactly three free variables.

b) A homogeneous linear system with no solution.

c) A 5×3 matrix in RREF such that $Ax = 0$ has a non-trivial solution.

7. Write an augmented matrix corresponding to a system of two linear equations in three variables x_1, x_2, x_3, whose solution set is the span of $\begin{pmatrix} -4 \\ 1 \\ 0 \end{pmatrix}$.

Briefly justify your answer.

8. Acme Widgets, Gizmos, and Doodads has two factories. Factory A makes 10 widgets, 3 gizmos, and 2 doodads every hour, and factory B makes 4 widgets, 1 gizmo, and 1 doodad every hour.

a) If factory A runs for a hours and factory B runs for b hours, how many widgets, gizmos, and doodads are produced? Express your answer as a vector equation.

b) A customer places an order for 16 widgets, 5 gizmos, and 3 doodads. Can Acme fill the order with no widgets, gizmos, or doodads left over? If so, how many hours do the factories run? If not, why not?

9. Consider the system below, where h and k are real numbers.

$$\begin{aligned} x + 3y &= 2 \\ 3x - hy &= k. \end{aligned}$$

a) Find the values of h and k which make the system inconsistent.

b) Find the values of h and k which give the system a unique solution.

c) Find the values of h and k which give the system infinitely many solutions.

10. Consider the following consistent system of linear equations.

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 &= -2 \\ 3x_1 + 4x_2 + 5x_3 + 6x_4 &= -2 \\ 5x_1 + 6x_2 + 7x_3 + 8x_4 &= -2 \end{aligned}$$

a) Find the parametric vector form for the general solution.

b) Find the parametric vector form of the corresponding *homogeneous* equations. [Hint: you've already done the work.]