Math 1553 Supplement §4.2, 4.3

1. Let A be a 3×4 matrix with column vectors v_1, v_2, v_3, v_4, and suppose $v_2 = 2v_1 - 3v_4$. Consider the matrix transformation $T(x) = Ax$.

a) Is it possible that T is one-to-one? If yes, justify why. If no, find distinct vectors v and w so that $T(v) = T(w)$.

b) Is it possible that T is onto? Justify your answer.

2. Which of the following transformations T are onto? Which are one-to-one? If the transformation is not onto, find a vector not in the range. If the matrix is not one-to-one, find two vectors with the same image.

a) The transformation $T : \mathbf{R}^3 \to \mathbf{R}^2$ defined by $T(x, y, z) = (y, y)$.

b) JUST FOR FUN: Consider T : (Smooth functions) $\to$ (Smooth functions) given by $T(f) = f'$ (the derivative of f). Then T is not a transformation from any $\mathbf{R}^n$ to $\mathbf{R}^m$, but it is still *linear* in the sense that for all smooth f and g and all scalars c (by properties of differentiation we learned in Calculus 1):

$$T(f+g) = T(f) + T(g) \quad \text{since} \quad (f+g)' = f' + g'$$
$$T(cf) = cT(f) \quad \text{since} \quad (cf)' = cf'.$$

Is T one-to-one?

3. In each case, determine whether T is linear. Briefly justify.

a) $T(x_1, x_2) = (x_1 - x_2, x_1 + x_2, 1)$.

b) $T(x, y) = (y, x^{1/3})$.

c) $T(x, y, z) = 2x - 5z$.

4. For each matrix A, describe what the associated matrix transformation T does to $\mathbf{R}^3$ geometrically.

a) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ **b)** $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

5. Let's go back to the 4.2-4.3 worksheet problem #3. The second little pig has decided to build his house out of sticks. His house is shaped like a pyramid with a triangular base that has vertices at the points $(0,0,0)$, $(2,0,0)$, $(0,2,0)$, and $(1,1,1)$.

The big bad wolf finds the pig's house and blows it down so that the house is rotated by an angle of 45° in a counterclockwise direction about the z-axis (look downward onto the xy- plane the way we usually picture the plane as $\mathbf{R}^2$), and then projected onto the xy-plane.

In the worksheet, we found the matrix for the transformation T caused by the wolf. Geometrically describe the image of the house under T.