Math 1553 Exam 1, SOLUTIONS, Fall 2024

Name		**GT ID**	

Circle your instructor and lecture below. Be sure to circle the correct choice!

Jankowski (A, 8:25-9:15) Wessels(B, 8:25-9:15) Hozumi (C, 9:30-10:20)

Wessels (D, 9:30-10:20) Kim (G, 12:30-1:20) Short (H, 12:30-1:20)

Shubin (I, 2:00-2:50) He (L, 3:30-4:20) Wan (M, 3:30-4:20)

Shubin (N, 5:00-5:50) Denton (W, 8:25-9:15)

Please read the following instructions carefully.

- Write your initials at the top of each page. The max score on this exam is 70 points, and you have 75 minutes to complete this exam. Each problem is worth 10 points.
- There are no calculators or aids of any kind (notes, text, etc.) allowed.
- As always, RREF means "reduced row echelon form."
- The "zero vector" in $\mathbf{R}^n$ is the vector in $\mathbf{R}^n$ whose entries are all zero.
- On free response problems, show your work, unless instructed otherwise. A correct answer without appropriate work may receive little or no credit!
- We will hand out loose scrap paper, but it **will not be graded** under any circumstances. All answers and work must be written on the exam itself, with no exceptions.
- This exam is double-sided. You should have more than enough space to do every problem on the exam, but if you need extra space, you may use the *back side of the very last page of the exam.* If you do this, you must clearly indicate it.
- You may cite any theorem proved in class or in the sections we covered in the text.
- For questions with bubbles to fill in, you must fill them in the correct bubbles clearly and completely or you will not receive credit.

I, the undersigned, hereby affirm that I will not share the contents of this exam with anyone. Furthermore, I have not received inappropriate assistance in the midst of nor prior to taking this exam. I will not discuss this exam with anyone in any form until after 7:45 PM on Wednesday, September 18.

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1. TRUE or FALSE. Clearly fill in the bubble for your answer. If the statement is *ever* false, fill in the bubble for False. You do not need to show any work, and there is no partial credit. Each question is worth 2 points.

   (a) If the RREF of an augmented matrix has bottom row $\left(\begin{array}{ccc|c} 0 & 1 & 0 & -5 \end{array}\right)$, then its corresponding system of linear equations must be consistent.

   ● True

   ○ False

   (b) If a consistent system of linear equations has more equations than variables, then the system must have exactly one solution.

   ○ True

   ● False

   (c) Suppose u, v, and w are vectors in $\mathbf{R}^3$ and $\text{Span}\{u, v, w\} = \mathbf{R}^3$. If b is any vector in $\mathbf{R}^3$, then the vector equation $x_1 u + x_2 v + x_3 w = b$ must have exactly one solution.

   ● True

   ○ False

   (d) There is a 2×2 matrix A so that the set of solutions to $A\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is the line $y = x - 1$.

   ○ True

   ● False

   (e) If A is an $m \times n$ matrix and $m > n$, then the homogeneous matrix equation $Ax = 0$ must have infinitely many solutions.

   ○ True

   ● False

Solution: Problem 1.

(a) True, since this means the rightmost column does not have a pivot.

(b) False: for example, consider the system for

$$\left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right).$$

(c) True. Since u, v, w are vectors in $\mathbf{R}^3$ and their span is $\mathbf{R}^3$, the 3×3 matrix with columns u, v, and w will have a pivot in every row and column, so any augmented matrix $\left(\begin{array}{ccc|c} u & v & w & b \end{array}\right)$ will row-reduce to the following general form no matter what b is, so we will get exactly one solution.

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & (*)_1 \\ 0 & 1 & 0 & (*)_2 \\ 0 & 0 & 1 & (*)_3 \end{array}\right).$$

(d) False: the equation $A \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is homogeneous, so it must have the trivial solution $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$. However, the line $y = x - 1$ does not contain $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

(e) False: for example, if $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$, then $Ax = 0$ has only the trivial solution $x = 0$.

2. **Full solutions are on the next page.**

(a) Which of the following matrices are in reduced row echelon form (RREF)? Clearly mark all that apply.

● $\left(\begin{array}{cccc|c} 1 & 6 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right)$ ● $\left(\begin{array}{cccc|c} 1 & -3 & 0 & 7 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right)$ ○ $\left(\begin{array}{cccc|c} 1 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$

(b) Suppose an augmented matrix with 3 rows and 5 columns (including its rightmost column) has 3 pivots and corresponds to a consistent system of linear equations. Clearly mark your answer for each of the following.

(i) The solution set is:
○ a point ● a line ○ a plane ○ $\mathbf{R}^3$ ○ $\mathbf{R}^4$

(ii) The solution set lives in:
○ $\mathbf{R}$ ○ $\mathbf{R}^2$ ○ $\mathbf{R}^3$ ● $\mathbf{R}^4$ ○ $\mathbf{R}^5$

(c) In each case, determine whether the equation in the variables x, y, and z is linear or not linear. Clearly mark your answers.

$3x - y - yz = 3$. ○ Linear ● Not linear

$4x + 11^{1/3}y + 3z = 2$. ● Linear ○ Not linear

(d) Suppose that the solution set to a matrix equation $Ax = b$ has parametric form

$$x_1 = 3 - x_3, \qquad x_2 = 1 + x_3, \qquad x_3 = x_3 \ (x_3 \text{ real}), \qquad x_4 = 0.$$

Which of the following statements **must** be true? Clearly mark all that apply.

● The vector $\begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$ is a solution to the homogeneous matrix equation $Ax = 0$.

○ b is a vector in $\mathbf{R}^4$.

● The matrix A has 4 columns.

Solution:

(a) The first two are in RREF. However, the third matrix is not in RREF because there is a "1" above the pivot in the fourth column.

(b) Since the augmented matrix has 5 columns including the rightmost column, there are 4 columns for the variables, therefore the solution set lives in $\mathbf{R}^4$. Since there are 3 pivots in the four left columns, this means we have $4-3=1$ free variable, so the solution set is a line.

(c) The first is not linear because of its yz term. The second is linear.

(d) For this problem, we use the fact that a solution set for a consistent system $Ax = b$ is obtained by taking one particular solution and adding all homogeneous solutions. If we write the solution set in parametric vector form, we will get

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 3 - x_3 \\ 1 + x_3 \\ x_3 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix}.$$

Thus, we see $\begin{pmatrix} 3 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ is a solution to $Ax = b$, and $\begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$ is a solution to $Ax = 0$.

i. Yes, the vector is a solution to $Ax = 0$ as described above.

ii. No, b is not necessarily in $\mathbf{R}^4$. For example, we could have

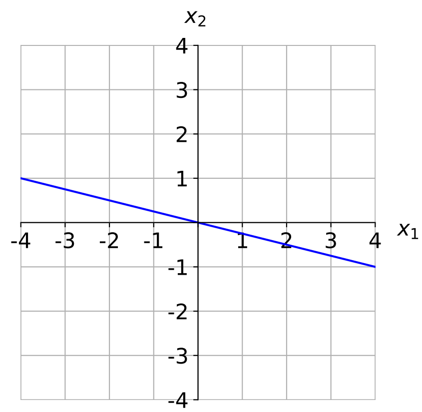
$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad b = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}.$$

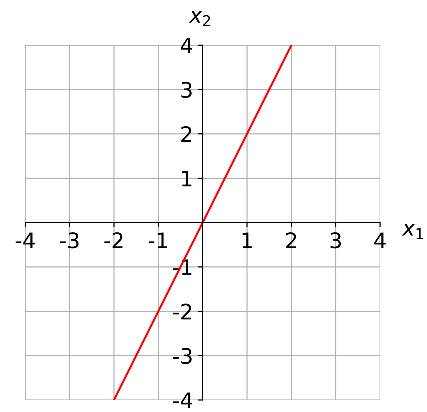
iii. Yes, A must have 4 columns because we were told that the solution set lives in $\mathbf{R}^4$.

3. Short answer and multiple choice. Parts (a) through (c) are unrelated. There is no partial credit except on (a).

(a) (4 points) For some 2×2 matrix A, the vector $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ is in the column span of A, and the vector $x = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ is a solution to $Ax = 0$.

On the **left** graph below, carefully draw the column span of A.
On the **right** graph below, carefully draw the solution set for $Ax = 0$.





(b) (2 points) Compute the product $\begin{pmatrix} 2 & -4 & -3 \\ 2 & 1 & 4 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$.

Select the correct answer below.

○ $\begin{pmatrix} 8 \\ 1 \end{pmatrix}$ ○ $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ ○ $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ ● $\begin{pmatrix} 8 \\ 3 \end{pmatrix}$ ○ $\begin{pmatrix} 8 \\ 3 \\ 0 \end{pmatrix}$

○ $\begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}$ ○ $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ ○ $\begin{pmatrix} 4 & 4 & 0 \\ 4 & -1 & 0 \end{pmatrix}$ ○ none of these

(c) (4 points) Suppose v_1, v_2, v_3, and b are vectors in $\mathbf{R}^n$ and that the augmented matrix $\left(\begin{array}{ccc|c} v_1 & v_2 & v_3 & b \end{array}\right)$ corresponds to a linear system of equations with infinitely many solutions. Which of the following statements are true? Clearly mark all that apply.

○ If A is the matrix with columns v_1, v_2, and v_3, then the span of the columns of A is $\mathbf{R}^3$.

● There are scalars x_1, x_2, and x_3 so that $x_1v_1 + x_2v_2 + x_3v_3 = b$.

● The vector b is in Span$\{v_1, v_2, v_3\}$.

● The vector equation $x_1v_1 + x_2v_2 + x_3v_3 = 0$ has infinitely many solutions.

Solution:

(a) The setup means that A has exactly one pivot column and exactly one column without a pivot, so the span of the columns of A and the solution set to $Ax = 0$ must both be lines. Also, spans always include the origin, and homogeneous solution sets always include the origin (since they always have the trivial solution $x = 0$).
Therefore, the column span of A is the line through the origin containing $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$, while the solution set to $Ax = 0$ is the line through the origin containing $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

(b) $\begin{pmatrix} 2 & -4 & -3 \\ 2 & 1 & 4 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} = 2 \begin{pmatrix} 2 \\ 2 \end{pmatrix} - \begin{pmatrix} -4 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ -1 \end{pmatrix} = \begin{pmatrix} 8 \\ 3 \end{pmatrix}.$

(c)
  i. This is not necessarily true. The columns of A might not even be vectors in $\mathbf{R}^3$.

  ii. True, this follows directly from the definition of linear combination since the system is consistent.

  iii. True, this follows directly from the definition of span since the system is consistent.

  iv. True: the vector equation with "b" has infinitely many solutions, and homogeneous solution sets are translations of non-homogeneous solution sets, so the same vector equation with "0" also has infinitely many solutions.

4. (a) Consider the system of linear equations corresponding to the augmented matrix $\left(\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 4 \end{array}\right)$. Which one of the following vectors is a solution to the system?

○ $\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ ● $\begin{pmatrix} -5 \\ -1 \\ 2 \end{pmatrix}$ ○ $\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$ ○ $\begin{pmatrix} -13 \\ -3 \\ 4 \end{pmatrix}$ ○ none of these

(b) In each case, determine if the span of the columns of the matrix is $\mathbf{R}^2$. Clearly mark your answer.

(i) $\begin{pmatrix} 4 & -8 & 1 \\ 8 & -16 & 2 \end{pmatrix}$ ○ Yes ● No

(ii) $\begin{pmatrix} 1 & 3 \\ -2 & 1 \\ 0 & 5 \end{pmatrix}$ ○ Yes ● No

(c) Suppose b is a vector in $\mathbf{R}^2$. Clearly mark which **one** of the following statements must be true about the vector equation: $x_1 \begin{pmatrix} 5 \\ 1 \end{pmatrix} + x_2 \begin{pmatrix} -2 \\ 2 \end{pmatrix} = b.$

○ We cannot conclude whether the vector equation is consistent unless we know the vector b.

● The vector equation must have exactly one solution, no matter what b is.

○ The vector equation must be consistent, but we cannot conclude whether it has one solution or infinitely many solutions unless we know what b is.

(d) Suppose that A is an $m \times n$ matrix and b is a vector so that the matrix equation $Ax = b$ is consistent. Which of the following statements are true? Clearly mark all that apply.

● The equation $Ax = b$ must be homogeneous if the zero vector is a solution.

○ If x is a solution to $Ax = b$, then x is a vector in $\mathbf{R}^m$.

● If the solution set to $Ax = b$ is a line, then the solution set to $Ax = 0$ must also be a line.

Solution:

(a) We can row-reduce or back-sub. The final row gives $x_3 = 2$, whereby the second row gives $x_2+2 = 1$, so $x_2 = -1$. The first row says $x_1-3(-1)+2 = 0$, so $x_1 = -5$. Therefore, the system has the unique solution $\begin{pmatrix} -5 \\ -1 \\ 2 \end{pmatrix}$.

(b) i. No, all three vectors are multiples of $\begin{pmatrix} 4 \\ 8 \end{pmatrix}$, so their span is just the span of $\begin{pmatrix} 4 \\ 8 \end{pmatrix}$, which is a line in $\mathbf{R}^2$. Another way to see this is that the matrix in question has only 1 pivot.

ii. No, neither vector is a scalar multiple of the other, so they span a plane in $\mathbf{R}^3$ (the matrix in question has two pivots). Recall that a plane in $\mathbf{R}^3$ is **not** the same thing as $\mathbf{R}^2$: every vector in $\mathbf{R}^2$ has exactly two entries, whereas every vector in $\operatorname{Span}\left\{\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix}\right\}$ has exactly 3 entries.

(c) The correct answer is (ii). Regardless of what b is, the matrix $\left(\begin{array}{cc|c} 5 & -2 & b_1 \\ 1 & 2 & b_2 \end{array}\right)$ will have two pivots to the left of the augment bar, therefore the corresponding vector equation will be consistent with exactly one solution.

(d) i. True: if $x = 0$ is a solution then $A(0) = b$ which means $0 = b$, so the system is homogeneous.

ii. No, x is in $\mathbf{R}^n$, so this is never true unless $m = n$.

iii. True, by the fundamental fact of section 2.4.

The rest of the exam is free response. Unless told otherwise, show your work! A correct answer without appropriate work will receive little or no credit, even it is correct. Parts (a) and (b) are unrelated.

5. (a) (5 pts) Solve the following linear system in the variables x_1, x_2, and x_3:

$$x_1 - x_2 - x_3 = 2$$

$$-x_1 + 2x_2 + 3x_3 = 0.$$

Write the solution set in parametric form.

Solution:

$$\left(\begin{array}{rrr|r} 1 & -1 & -1 & 2 \\ -1 & 2 & 3 & 0 \end{array}\right) \xrightarrow{R_2 = R_2 + R_1} \left(\begin{array}{rrr|r} \boxed{1} & -1 & -1 & 2 \\ 0 & \boxed{1} & 2 & 2 \end{array}\right) \xrightarrow{R_1 = R_1 + R_2} \left(\begin{array}{rrr|r} \boxed{1} & 0 & 1 & 4 \\ 0 & \boxed{1} & 2 & 2 \end{array}\right).$$

So $x_1 + x_3 = 4$ and $x_2 + 2x_3 = 2$ where x_3 is free. The parametric form of the solution set is given below. Any way in which the student articulates that x_3 is free is fine.

$$x_1 = 4 - x_3, \qquad x_2 = 2 - 2x_3, \qquad x_3 = x_3 \ (x_3 \text{ real}).$$

(b) (5 points) Find all values of h and k so that the following system has exactly one solution:

$$2x - hy = 5$$

$$6x + 9y = k.$$

Solution: We need pivots in both of the two columns to the left of the augment bar in order for the system to have exactly one solution:

$$\left(\begin{array}{rr|r} \boxed{2} & -h & 5 \\ 6 & 9 & k \end{array}\right) \xrightarrow{R_2 = R_2 - 3R_1} \left(\begin{array}{rr|r} \boxed{2} & -h & 5 \\ 0 & \boxed{9+3h} & k-15 \end{array}\right).$$

Since $9 + 3h$ must be a pivot spot, we need $9 + 3h \neq 0$, so $\boxed{h \neq -3}$. With this satisfied, it is irrelevant what the $k - 15$ entry is, since all pivots are already established and we are guaranteed a unique solution: k can be any real number.

Free response. Unless told otherwise, show your work! A correct answer without appropriate work will receive little or no credit, even it is correct.

6. Consider the following linear system of equations in the variables $x_1,\ x_2,\ x_3,\ x_4$:

$$x_1 - 3x_2 + x_3 + x_4 = 3$$

$$2x_1 - 6x_2 + 3x_3 + 2x_4 = 3$$

$$-3x_1 + 9x_2 - 3x_3 - 2x_4 = -8.$$

(a) (5 points) Write the augmented matrix corresponding to this system, and put the augmented matrix into RREF.

Solution:

$$\left(\begin{array}{rrrr|r} 1 & -3 & 1 & 1 & 3 \\ 2 & -6 & 3 & 2 & 3 \\ -3 & 9 & -3 & -2 & -8 \end{array}\right) \xrightarrow[R_3=R_3+3R_1]{R_2=R_2-2R_1} \left(\begin{array}{rrrr|r} 1 & -3 & 1 & 1 & 3 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right)$$

$$\xrightarrow{R_1=R_1-R_3} \left(\begin{array}{rrrr|r} 1 & -3 & 1 & 1 & 2 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right)$$

$$\xrightarrow{R_1=R_1-R_2} \left(\begin{array}{rrrr|r} \boxed{1} & -3 & 0 & 0 & 5 \\ 0 & 0 & \boxed{1} & 0 & -3 \\ 0 & 0 & 0 & \boxed{1} & 1 \end{array}\right)$$

(b) (4 points) The system is consistent. Write the set of solutions to the system of equations in parametric vector form.

Solution: From (a) we have $x_1 - 3x_2 = 5$ (so $x_1 = 5 + 3x_2$), x_2 is free, $x_3 = -3$, and $x_4 = 1$. In parametric vector form:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 5+3x_2 \\ x_2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ -3 \\ 1 \end{pmatrix} + x_2 \begin{pmatrix} 3 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ -3 \\ 1 \end{pmatrix} + x_2 \begin{pmatrix} 3 \\ 1 \\ 0 \\ 0 \end{pmatrix}.$$

(c) (1 pt) Write *one* vector x that is not the zero vector and solves the corresponding **homogeneous** system below. There is no partial credit on this part, so take time to check by hand that your answer is correct, and if it is not correct then go back and check your work from above!

$$\begin{aligned} x_1 - 3x_2 + x_3 + x_4 &= 0 \\ 2x_1 - 6x_2 + 3x_3 + 2x_4 &= 0 \\ -3x_1 + 9x_2 - 3x_3 - 2x_4 &= 0. \end{aligned} \qquad x = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

Solution: From the parametric vector form above, we see $\begin{pmatrix} 3 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ is a solution to the corresponding homogeneous system.

Therefore, the correct answer is any nonzero multiple of $\begin{pmatrix} 3 \\ 1 \\ 0 \\ 0 \end{pmatrix}$.

The question emphasizes checking your answer to make sure it works, so let's check and see:

$$3 - 3(1) + 0 + 0 = 0$$
$$2(3) - 6(1) + 0 + 0 = 0$$
$$-3(3) + 9(1) - 0 - 0 = 0.$$

Side note: you could also take a moment to check the other part of the answer in part (b) by plugging $\begin{pmatrix} 5 \\ 0 \\ -3 \\ 1 \end{pmatrix}$ into the original system:

$$5 - 3(0) - 3 + 1 = 3$$
$$2(5) - 6(0) + 3(-3) + 2 = 3$$
$$-3(5) + 9(0) - 3(-3) - 2 = -8.$$

Free response. Unless told otherwise, show your work! A correct answer without appropriate work will receive little or no credit, even if the answer is correct. Parts (a) and (b) are unrelated.

7. (a) Find all values of h (if there are any) so that the vector $\begin{pmatrix} 9 \\ h \\ -7 \end{pmatrix}$ can be written as a linear combination of $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}$.

Solution: We set up an augmented matrix and row-reduce to solve for when the vector equation $x_1 \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + x_2 \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 9 \\ h \\ -7 \end{pmatrix}$ is consistent.

$$\left(\begin{array}{cc|c} 1 & -3 & 9 \\ 2 & 1 & h \\ -1 & 2 & -7 \end{array}\right) \xrightarrow[R_3 = R_3 + R_1]{R_2 = R_2 - 2R_1} \left(\begin{array}{cc|c} 1 & -3 & 9 \\ 0 & 7 & h-18 \\ 0 & -1 & 2 \end{array}\right) \xrightarrow{R_2 \leftrightarrow R_3} \left(\begin{array}{cc|c} 1 & -3 & 9 \\ 0 & -1 & 2 \\ 0 & 7 & h-18 \end{array}\right)$$

$$\xrightarrow{R_3 = R_3 + 7R_2} \left(\begin{array}{cc|c} 1 & -3 & 9 \\ 0 & -1 & 2 \\ 0 & 0 & h-4 \end{array}\right).$$

This system is consistent if and only if the rightmost column is not a pivot column, therefore $h - 4 = 0$, so $\boxed{h = 4}$.

(b) Write an augmented matrix in reduced row echelon form, so that the set of solutions to the corresponding system of equations has parametric form given below. You do not need to show your work on this part.

$$x_1 = -3x_3, \qquad x_2 = 0, \qquad x_3 = x_3 \ (x_3 \text{ real}).$$

Solution: Unraveling the first equation gives $x_1 + 3x_3 = 0$, so one row could be $\left(\begin{array}{ccc|c} 1 & 0 & 3 & 0 \end{array}\right)$. We also need $x_2 = 0$, so our second row could be $\left(\begin{array}{ccc|c} 0 & 1 & 0 & 0 \end{array}\right)$. Possible answers are

$$\left(\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right), \quad \left(\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right), \quad \left(\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right), \quad \text{etc..}$$

This page is reserved ONLY for work that did not fit elsewhere on the exam.

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