Math 1553 Exam 1, Spring 2025

Name		GT ID	

Circle your instructor and lecture below. Be sure to circle the correct choice!

Jankowski (A+HP, 8:25-9:15 AM) Jankowski (C, 9:30-10:20 AM)

Al Ahmadieh (I, 2:00-2:50 PM) Al Ahmadieh (M, 3:30-4:20 PM)

Please read the following instructions carefully.

- Write your initials at the top of each page. The maximum score on this exam is 70 points, and you have 75 minutes to complete it. Each problem is worth 10 points.
- There are no calculators or aids of any kind (notes, text, etc.) allowed.
- As always, RREF means "reduced row echelon form." The "zero vector" in $\mathbf{R}^n$ is the vector in $\mathbf{R}^n$ whose entries are all zero.
- On free response problems, show your work, unless instructed otherwise. A correct answer without appropriate work may receive little or no credit!
- We will hand out loose scrap paper, but it **will not be graded** under any circumstances. All answers and work must be written on the exam itself, with no exceptions.
- This exam is double-sided. You should have more than enough space to do every problem on the exam, but if you need extra space, you may use the *back side of the very last page of the exam.* If you do this, you must clearly indicate it.
- You may cite any theorem proved in class or in the sections we covered in the text.
- For questions with bubbles, either fill in the bubble completely or leave it blank. **Do not** mark any bubble with "X" or "/" or any such intermediate marking. Anything other than a blank or filled bubble may result in a 0 on the problem, and regrade requests may be rejected without consideration.

I, the undersigned, hereby affirm that I will not share the contents of this exam with anyone. Furthermore, I have not received inappropriate assistance in the midst of nor prior to taking this exam. I will not discuss this exam with anyone in any form until after 7:45 PM on Wednesday, February 5.

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1. TRUE or FALSE. Clearly fill in the bubble for your answer. If the statement is *ever* false, fill in the bubble for False. You do not need to show any work, and there is no partial credit. Each question is worth 2 points.

   (a) If an augmented matrix in reduced row echelon form (RREF) has a pivot in every row, then the corresponding system of linear equations must have exactly one solution.

   ○ True

   ○ False

   (b) The vector equation below is consistent for each b in $\mathbf{R}^2$:

$$x_1 \begin{pmatrix} 1 \\ 3 \end{pmatrix} + x_2 \begin{pmatrix} 4 \\ 4 \end{pmatrix} = b.$$

   ○ True

   ○ False

   (c) Suppose u, v, and w are vectors in $\mathbf{R}^3$ with the property that Span$\{u, v\}$ is a plane and Span$\{u, w\}$ is a plane. Then Span$\{v, w\}$ must also be a plane.

   ○ True

   ○ False

   (d) If A is a 2×3 matrix and the solution set for $Ax = 0$ is a line, then every vector in $\mathbf{R}^2$ is in the span of the columns of A.

   ○ True

   ○ False

   (e) Suppose A is a 3×3 matrix and $A \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

   Then the matrix equation $Ax = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ must have infinitely many solutions.

   ○ True

   ○ False

2. On this page, you do not need to show work and there is no partial credit. Parts (a) through (d) are unrelated.

(a) (3 points) Which of the following equations are linear equations in the variables x, y, and z? Clearly fill in the bubble for all that apply.

○ $x + \sin\left(\frac{\pi}{7}\right) y + 10z = -2$

○ $x - yz = 1$

○ $x - y + z = 3$

(b) (3 points) Which of the following matrices are in reduced row echelon form? Clearly fill in the bubble for all that apply.

○ $\left(\begin{array}{cccc|c} 1 & 0 & 2 & 3 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$

○ $\left(\begin{array}{cccc|c} 0 & 0 & 1 & 5 & -1 \end{array}\right)$

○ $\left(\begin{array}{cccc|c} 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right)$

(c) (2 pts) Suppose a **consistent** linear system corresponds to an augmented matrix with 7 rows, 5 total columns (including its rightmost column), and 3 pivots in its RREF.

(i) Geometrically, what is the solution set for the system of equations? Clearly fill in the **one** correct bubble for your answer.

○ no solutions ○ a point ○ a line ○ a plane

○ all of $\mathbf{R}^2$ ○ all of $\mathbf{R}^3$

(ii) Where does the solution set live?

○ $\mathbf{R}^3$ ○ $\mathbf{R}^4$ ○ $\mathbf{R}^5$ ○ $\mathbf{R}^6$ ○ $\mathbf{R}^7$

(d) (2 points) Compute $\begin{pmatrix} 1 & 2 \\ 0 & 2 \\ -1 & 5 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \end{pmatrix}$. Clearly fill in the bubble for your answer.

○ $\begin{pmatrix} 3 \\ 4 \\ 11 \end{pmatrix}$ ○ $\begin{pmatrix} -1 & 4 \\ 0 & 4 \\ 1 & 10 \end{pmatrix}$ ○ $\begin{pmatrix} 5 \\ 4 \\ 9 \end{pmatrix}$ ○ $\begin{pmatrix} 3 \\ 0 \\ 11 \end{pmatrix}$ ○ none of these

3. On this page, you do not need to show work and there is no partial credit. Parts (a) through (d) are unrelated.

(a) (2 points) Consider the vector equation $x_1 \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$.

Which **one** of the following describes its solution set? Clearly fill in the bubble for your answer.

○ no solutions ○ a point in $\mathbf{R}^2$ ○ a line in $\mathbf{R}^2$ ○ all of $\mathbf{R}^2$

○ a point in $\mathbf{R}^3$ ○ a line in $\mathbf{R}^3$ ○ a plane in $\mathbf{R}^3$ ○ all of $\mathbf{R}^3$

(b) (2 points) Suppose v_1, v_2, and b are vectors in $\mathbf{R}^3$ with the properties that $\text{Span}\{v_1, v_2\}$ is a line and b is **not** in $\text{Span}\{v_1, v_2\}$. Which of the following statements are true? Clearly fill in the bubble for all that apply.

○ The vector equation $x_1v_1 + x_2v_2 = b$ is inconsistent.

○ If w is a vector in $\mathbf{R}^3$ and the vector equation $x_1v_1 + x_2v_2 = w$ is consistent, then it must have infinitely many solutions.

(c) (4 pts) Suppose the solution set to some matrix equation $Ax = b$ has parametric vector form given below, where x_3 is free:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}.$$

(i) Which of these statements must be true? Fill in the bubble for all that apply.

○ The matrix equation $Ax = b$ is not homogeneous.

○ The matrix A has 3 rows.

(ii) Which **one** of the following vectors is a solution to $Ax = 0$?

○ $\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$ ○ $\begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$

(d) (2 points) Find all values of h (if there are any) so that $\begin{pmatrix} 3 \\ h \end{pmatrix}$ can be written as a linear combination of $\begin{pmatrix} 1 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 10 \end{pmatrix}$.

○ $h = 0$ only ○ $h = 5$ only ○ $h = 10$ only ○ $h = -10$ only

○ $h = 15$ only ○ $h = -15$ only ○ all real h ○ none of these

4. On this page, you do not need to show your work. Only your answers are graded. Parts (a)-(c) are unrelated.

(a) (3 points) Suppose A is an $m \times n$ matrix and b is in $\mathbf{R}^m$. Which of the following conditions **guarantee** that $Ax = b$ is consistent? Clearly fill in the bubble for all that apply.

- ◯ b is in the span of the columns of A.
- ◯ A has a pivot in every column.
- ◯ The augmented matrix $\left(A \mid b \right)$ has a pivot in every row.

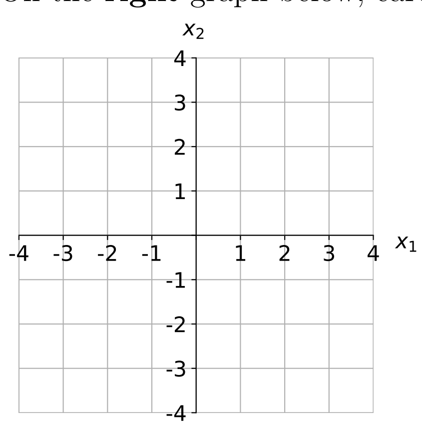
(b) (3 points) Write a set of 3 **different** vectors $\{v_1, v_2, v_3\}$ in $\mathbf{R}^3$ so that Span$\{v_1, v_2, v_3\}$ is a line.

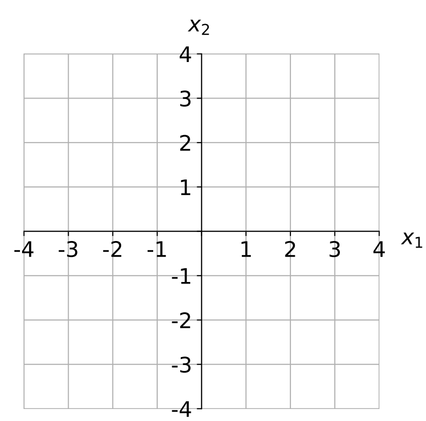
$$v_1 = \begin{pmatrix} \\ \\ \\ \end{pmatrix} \qquad v_2 = \begin{pmatrix} \\ \\ \\ \end{pmatrix} \qquad v_3 = \begin{pmatrix} \\ \\ \\ \end{pmatrix}.$$

(c) (4 points) Suppose b is a nonzero vector in $\mathbf{R}^2$ and A is a 2×2 matrix that satisfies the following conditions:

- The vector $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ is a solution to $Ax = 0$.
- The vector $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ is a solution to $Ax = b$.

On the **left** graph below, carefully draw the solution set to $Ax = 0$.
On the **right** graph below, carefully draw the solution set to $Ax = b$.





5. Free response. Show your work! A correct answer without appropriate work will receive little or no credit. For the row-reduction steps you put in part (a) of the problem, you do not need to repeat them in parts (b) and (c).

Consider the system of linear equations in x, y, and z given below:

$$x + y = 0$$

$$y + z = 0$$

$$x + 2cz = 1.$$

(a) Determine all values of c (if there are any) so that the system of equations is inconsistent.

(b) Determine all values of c (if there are any) so that the system of equations has exactly one solution.

(c) Determine all values of c (if there are any) so that the system of equations has infinitely many solutions.

6. Free response. Show your work unless otherwise indicated! A correct answer without sufficient work may receive little or no credit.
Consider the following linear system of equations in the variables $x_1,\ x_2,\ x_3,\ x_4$:

$$x_1 - x_2 - 2x_3 = -2$$

$$2x_1 - 2x_2 - 4x_3 + 2x_4 = -8$$

$$-2x_1 + 3x_2 + 4x_3 = 7.$$

(a) (5 points) Write the augmented matrix corresponding to this system, and put the augmented matrix into RREF.

(b) (4 points) The system is consistent. Write the set of solutions to the system of equations in parametric **vector** form.

(c) (1 pt) Write ***one*** vector x that solves the system of equations. There is no partial credit on this part, so take time to check by hand that your answer is correct, and if it is not correct then check your work above! If you write more than one vector on this part, or if your answer is unclear, you will not receive any credit.

7. Free response. Show your work! A correct answer without sufficient work will receive little or no credit. Parts (a) and (b) are unrelated.

   (a) (5 points) Find all solutions to the matrix equation $\begin{pmatrix} 1 & -2 & 2 \\ 1 & 1 & 2 \end{pmatrix} x = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$.
   Write your answer in parametric form.

   (b) (5 points) Write an augmented matrix in RREF that corresponds to a system of linear equations in the variables x_1, x_2, and x_3 whose solution set has parametric form

   $$x_1 = 1 - 3x_3 \qquad x_2 = 2x_3 \qquad x_3 = x_3 \quad (x_3 \text{ any real number}).$$

   Briefly justify why your matrix satisfies these conditions.

This page is reserved ONLY for work that did not fit elsewhere on the exam.

If you use this page, please clearly indicate (on the problem's page and here) which problems you are doing.