MATH 1553, JANKOWSKI
MIDTERM 3, SPRING 2018, LECTURE A

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Write your section number here: ____________

Please **read all instructions** carefully before beginning.

- Please leave your GT ID card on your desk until your TA matches your exam.
- The maximum score on this exam is 50 points.
- You have 50 minutes to complete this exam.
- There are no aids of any kind (calculators, notes, text, etc.) allowed.
- Show your work unless specified otherwise. A correct answer without appropriate work will receive little or no credit.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

Problem 1.

[2 points each]

On problem 1, you do not need to justify your answer, and there is no partial credit.

a) Write a 2×2 matrix A which is invertible but not diagonalizable.

The remaining problems are true or false. Answer true if the statement is *always* true. Otherwise, answer false. In every case, assume that the entries of the matrix A are real numbers.

b) **T** **F** If A is the 3×3 matrix satisfying $Ae_1 = e_2$, $Ae_2 = e_3$, and $Ae_3 = e_1$, then $\det(A) = 1$.

c) **T** **F** If A is an $n \times n$ matrix and $\det(A) = 2$, then 2 is an eigenvalue of A.

d) **T** **F** The matrices $A = \begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 2 \\ 0 & -1 \end{pmatrix}$ are similar.

e) **T** **F** If A is an $n \times n$ matrix and v and w are eigenvectors of A, then $v + w$ is also an eigenvector of A.

f) **T** **F** If A is an invertible $n \times n$ matrix and B is similar to A, then B is invertible.

Extra space for scratch work on problem 1

Problem 2.

[8 points]

Short answer. Show your work on part (b). In every case, the entries of each matrix must be real numbers.

a) Write a 2×2 matrix A for which $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are eigenvectors corresponding to the same eigenvalue.

b) Find the area of the triangle with vertices $(0,0)$, $(1,4)$, and $(4,2)$.

c) Write a 3×3 matrix A with only one real eigenvalue $\lambda = 4$, such that the 4-eigenspace for A is a two-dimensional plane in $\mathbf{R}^3$.

d) Suppose A is an $n \times n$ matrix. Which of the following must be true? Circle all that apply.

I. If $\det(A) = 0$ then A is not invertible.

II. If A is diagonalizable, then A has n distinct eigenvalues.

Extra space for work on problem 2

Problem 3.

[10 points]

Consider the matrix

$$A = \begin{pmatrix} 3 & -7 \\ 1 & -1 \end{pmatrix}$$

a) Find all eigenvalues of A. Simplify your answer.

b) For the eigenvalue with negative imaginary part, find an eigenvector.

c) Using the eigenvalue with negative imaginary part, find a matrix C that is a composition of of rotation and scaling and which is similar to A.

d) Write the scale factor for C.

e) By what counterclockwise angle does your matrix C rotate? Simplify your answer (don't leave it in terms of arctan), as it is a standard angle.

Extra space for work on problem 3

Problem 4.

[10 points]

Let $A = \begin{pmatrix} -1 & 0 & -2 \\ 0 & 2 & 0 \\ 3 & 0 & 4 \end{pmatrix}$.

a) Find the eigenvalues of A.

b) Find a basis for each eigenspace of A. Mark your answers clearly.

c) Is A diagonalizable? If your answer is yes, find a diagonal matrix D and an invertible matrix P so that $A = PDP^{-1}$. If your answer is no, justify why A is not diagonalizable.

Extra space for work on problem 4

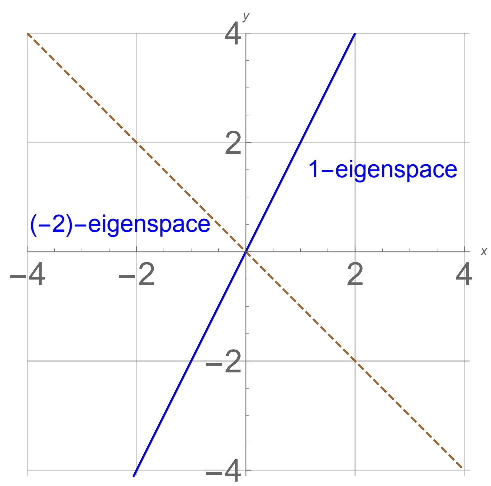
Problem 5.

[10 points]

Parts (a) and (b) are not related.

a) Find $\det(A^3)$ if $A = \begin{pmatrix} 1 & -3 & 4 & 2 \\ 0 & 0 & -2 & 0 \\ 0 & 1 & 2 & 3 \\ 2 & 0 & -1 & 20 \end{pmatrix}$.

b) Find the 2×2 matrix A whose eigenspaces are drawn below. Fully simplify your answer. (to be clear: the dashed line is the (-2)-eigenspace).



Extra space for work on problem 5