MATH 1553, JANKOWSKI
MIDTERM 3, FALL 2019

Name		**Section**	

Please **read all instructions** carefully before beginning.

- Write your name on the front of each page (not just the cover page!).
- The maximum score on this exam is 50 points, and you have 50 minutes to complete this exam.
- There are no calculators or aids of any kind (notes, text, etc.) allowed.
- As always, RREF means "reduced row echelon form."
- As always, $e_1, \ldots, e_n$ are the standard unit coordinate vectors in $\mathbf{R}^n$.
- Show your work, unless instructed otherwise. A correct answer without appropriate work will receive little or no credit! If you cannot fit your work on the front side of the page. use the back side of the page and indicate that you are using the back side.
- We will hand out loose scrap paper, but it **will not be graded** under any circumstances. All work must be written on the exam itself.
- You may cite any theorem proved in class or in the sections we covered the text.
- Good luck!

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Problem 1. [10 points]

These problems are true or false. Circle **T** if the statement is *always* true. Otherwise, circle **F**. You do not need to justify your answer. Assume that all entries of matrices A and B are real numbers.

a) **T** **F** If A is a 3×3 matrix and its characteristic polynomial is $-\lambda^3 + 2\lambda^2 - 17\lambda$, then $\det(A) = 0$.

b) **T** **F** Suppose A is a 3×3 matrix with columns v_1, v_2, v_3. If $v_1 - v_2 + v_3 = 0$, then the determinant of A must be zero.

c) **T** **F** Suppose A and B are $n \times n$ matrices with the same characteristic polynomial. If A is diagonalizable, then B must also be diagonalizable.

d) **T** **F** Suppose A and B are $n \times n$ matrices. If $\det(A) = \det(B)$, then $\det(A-B) = 0$.

e) **T** **F** Suppose A is a 3×3 matrix and that v is an eigenvector of A corresponding to the eigenvalue $\lambda = -1$. Then $-2v$ must also be an eigenvector of A corresponding to the eigenvalue $\lambda = -1$.

Extra space for scratch work on problem 1

Problem 2. [12 points]

Short answer. All parts are unrelated. You do not need to show your work. In each part, all entries of all matrices are real numbers.

a) Suppose $\det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = 2$. Fill in the blank:

$$\det\begin{pmatrix} a & b & c \\ g & h & i \\ -2d+3g & -2e+3h & -2f+3i \end{pmatrix} = \underline{\hspace{3cm}}.$$

b) Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the transformation that reflects across the line $y = -2x$, and let A be the standard matrix for T. Which of the following are true? Circle all that apply.

(i) The 1-eigenspace of A is $\text{Span}\left\{\begin{pmatrix} 1 \\ -2 \end{pmatrix}\right\}$.

(ii) A is diagonalizable.

(iii) $\det(A+I) = 0$.

c) Write a 2×2 matrix A that is diagonalizable but not invertible.

d) Suppose A is a 3×3 matrix and its characteristic polynomial is

$$\det(A - \lambda I) = -(\lambda - 5)(\lambda - 3)^2.$$

Which of the following *must* be true? Circle all that apply.

(i) The 5-eigenspace of A has dimension 1.

(ii) The homogeneous system given by the equation $(A - 3I)x = 0$ has two free variables.

(iii) For each b in $\mathbf{R}^3$, the equation $Ax = b$ is consistent.

e) Is there a 3×3 matrix A with the property that the 2-eigenspace of A is $\mathbf{R}^3$? If your answer is yes, write such a matrix A.

Extra space for work on problem 2

Problem 3.

[8 points]

Let $A = \begin{pmatrix} 2 & 5 \\ -1 & 4 \end{pmatrix}$.

a) Find the characteristic polynomial of A and the complex eigenvalues of A. Simplify your eigenvalues completely.

b) For the eigenvalue of A with *negative* imaginary part, find a corresponding eigenvector v.

c) Using your answer from (b), write an eigenvector w of A corresponding to the eigenvalue with *positive* imaginary part. You do not need to show your work on this part.

Extra space for work on problem 3

Problem 4.

[10 points]

Let $A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 3 & -2 \\ 0 & 2 & -2 \end{pmatrix}$.

a) Find the eigenvalues of A.

b) For each eigenvalue of A, find a basis for the corresponding eigenspace.

c) Is A diagonalizable? If your answer is yes, write an invertible 3×3 matrix C and a diagonal matrix D so that $A = CDC^{-1}$. If your answer is no, justify why A is not diagonalizable.

Extra space for work on problem 4

Problem 5.

[10 points]

Parts (a) and (b) are unrelated.

a) Suppose A and B are 4×4 matrices satisfying

$$\det(A) = 5, \quad \det(AB^{-1}) = 10.$$

Find $\det(-2B)$. Simplify your answer completely.

b) Let $A = C\begin{pmatrix} -1 & 0 \\ 0 & 1/2 \end{pmatrix}C^{-1}$, where the columns of C are (in order) $v_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $v_2 = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$.

(i) Write one nonzero vector v so that $A^n v$ approaches the zero vector as n gets very large (you do not need to show your work on part (i)).

(ii) Find $A^{10}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Show your work!

(iii) Clearly circle your answer: Is A^4 diagonalizable? YES NO
(no justification required for (iii))

Extra space for work on problem 5