MATH 1553, C. JANKOWSKI
MIDTERM 3

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Write your section number (E6-E9) here: ____________

Please **read all instructions** carefully before beginning.

- Please leave your GT ID card on your desk until your TA matches your exam.
- The maximum score on this exam is 50 points.
- You have 50 minutes to complete this exam.
- There are no aids of any kind (notes, text, etc.) allowed.
- Please show your work. If you cannot fit your work on the front side of the page, use the back side of the page as indicated.
- We will hand out loose scrap paper, but it **will not be graded** under any circumstances. All work must be written on the exam itself.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

Problem 1.

[Parts a) through f) are worth 2 points each]

a) Suppose A is a 3×3 matrix whose entries are real numbers. How many *distinct* real eigenvalues can A possibly have? Circle all that apply.

(a) 0

(b) 1

(c) 2

(d) 3

The remaining problems are true or false. Answer true if the statement is *always* true. Otherwise, answer false. You do not need to justify your answer. In every case, assume that the entries of the matrix A are real numbers.

b) **T** **F** If A is an $n \times n$ matrix then $\det(-A) = -\det(A)$.

c) **T** **F** If v is an eigenvector of a square matrix A, then $-v$ is also an eigenvector of A.

d) **T** **F** If A is an $n \times n$ matrix and $\lambda = 2$ is an eigenvalue of A, then $\mathrm{Nul}(A - 2I) = \{0\}$.

e) **T** **F** If A is a 3×3 matrix with characteristic polynomial $(3-\lambda)^2(2-\lambda)$, then the eigenvalue $\lambda = 2$ must have geometric multiplicity 1.

f) **T** **F** The matrix $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$ is similar to $\begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$.

Extra space for scratch work on problem 1

Problem 2.

[10 points]

Short answer. For (a) and (b), show any brief computations. For (c), (d), and (e), you do not need to justify your answer. In each case, assume the entries of A and B are real numbers.

a) Let $A = \begin{pmatrix} -1 & 1 \\ 1 & 7 \end{pmatrix}$, and define a transformation $T : \mathbf{R}^2 \to \mathbf{R}^2$ by $T(x) = Ax$. Find the area of $T(S)$, if S is a triangle in $\mathbf{R}^2$ with area 2.

b) Find $\det \begin{pmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 3 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 5 & 0 \end{pmatrix}$.

c) Write a 2×2 matrix A which is not diagonalizable and not invertible.

d) Give an example of 2×2 matrices A and B which have the same characteristic polynomial but are not similar.

e) Write a diagonalizable 3×3 matrix A whose only eigenvalue is $\lambda = 2$.

Extra space for work on problem 2

Problem 3.

[10 points]

Let $A = \begin{pmatrix} 2 & -4 \\ 1 & 2 \end{pmatrix}$.

a) Find the eigenvalues of A.

b) Let λ be the eigenvalue of A whose imaginary part is negative. Find an eigenvector of A corresponding to λ.

c) Find a matrix C which is similar to A and represents a composition of scaling and rotation.

d) What is the scaling factor for C?

e) Find the angle of rotation for C.
(do not leave your answer in terms of arctan; the answer is a standard angle).

Extra space for work on problem 3

Problem 4. [9 points]

$$A = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 2 & 4 \\ 0 & 0 & -1 \end{pmatrix}.$$

a) Find the eigenvalues of A, and find a basis for each eigenspace.

b) Is A diagonalizable? If your answer is yes, find a diagonal matrix D and an invertible matrix P so that $A = PDP^{-1}$. If your answer is no, justify why A is not diagonalizable.

Extra space for work on problem 4

Problem 5. [9 points]

Parts (a) and (b) are not related.

a) Find a 2×2 matrix A whose 2-eigenspace is the line $y = 2x$ and whose (-1)-eigenspace is the line $y = 3x$. Be sure your work is clear.

b) Let B be a 4×4 matrix satisfying $\det(B) = 2$, and let

$$C = \begin{pmatrix} 2 & 0 & 1 & 2 \\ 0 & 0 & 2 & 3 \\ -1 & 1 & 3 & 4 \\ 0 & 0 & 1 & -1 \end{pmatrix}.$$

Find $\det(CB^{-1})$.

Extra space for work on problem 5