MATH 1553, JANKOWSKI
MIDTERM 2, SPRING 2018, LECTURE A

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Write your section number here: ____________

Please **read all instructions** carefully before beginning.

- Please leave your GT ID card on your desk until your TA matches your exam.
- The maximum score on this exam is 50 points.
- You have 50 minutes to complete this exam.
- There are no aids of any kind (calculators, notes, text, etc.) allowed.
- Please show your work unless instructed otherwise. A correct answer without appropriate work will receive little or no credit. If you cannot fit your work on the front side of the page, use the back side of the page as indicated.
- We will hand out loose scrap paper, but it **will not be graded** under any circumstances. All work must be written on the exam itself.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

Problem 1.

[2 points for each part]

These problems are true or false. Circle **T** if the statement is *always* true. Otherwise, answer **F**. You do not need to justify your answer.

a) **T** **F** If A is an $n \times n$ matrix and $Ax = 0$ has only the trivial solution, then the equation $Ax = b$ is consistent for every b in $\mathbf{R}^n$.

b) **T** **F** If a matrix A has more columns than rows, then the linear transformation T given by $T(x) = Ax$ is not one-to-one.

c) **T** **F** If A and B are 3×3 matrices and the columns of B are linearly dependent, then the columns of AB are linearly dependent.

d) **T** **F** There are linear transformations $T : \mathbf{R}^4 \to \mathbf{R}^3$ and $U : \mathbf{R}^3 \to \mathbf{R}^4$ so that $T \circ U$ is invertible.

e) **T** **F** If a set S of vectors contains fewer vectors than there are entries in the vectors, then the set is linearly independent.

Extra space for scratch work on problem 1

Problem 2.

[12 points]

Parts (a) to (d) are unrelated. You do not need to justify answers in (a) or (b).

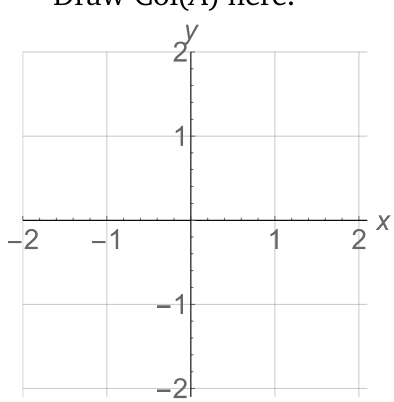
a) Write three different nonzero vectors v_1, v_2, v_3 in $\mathbf{R}^3$ so that $\{v_1, v_2, v_3\}$ is linearly dependent but v_3 is not in $\mathrm{Span}\{v_1, v_2\}$. Clearly indicate which vector is v_3.

b) Fill in the blanks: If A is a 5×6 matrix and its column span has dimension 2, then the null space of A is a ________-dimensional subspace of $\mathbf{R}^{\square}$.

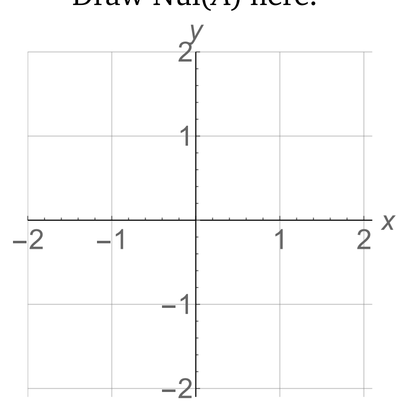
c) Find the matrix A satisfying $A^{-1}e_1 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$, $A^{-1}e_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, and $A^{-1}e_3 = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$.

d) Let $A = \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}$. Clearly draw Col($A$) and Nul($A$). Briefly show work.

Draw Col(A) here.



Draw Nul(A) here.



Extra space for work on problem 2

Problem 3. [10 points]

Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the transformation of reflection about the line $y = x$, and let $U : \mathbf{R}^2 \to \mathbf{R}^3$ be the transformation $U\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x - y \\ x \\ 3y \end{pmatrix}$.

a) Write the standard matrix A for T. Is T onto?

b) Write the standard matrix B for U. Is U one-to-one? Briefly justify your answer.

c) Circle the composition that makes sense: $T \circ U$ $\qquad$ $U \circ T$

d) Compute the standard matrix for the composition you circled in part (c).

Extra space for work on problem 3

Problem 4.

[10 points]

Dino McBarker has put the matrix A below in its reduced row echelon form:

$$A = \begin{pmatrix} 1 & -2 & 0 & 4 \\ -7 & 14 & 3 & 2 \\ 4 & -8 & -2 & -4 \end{pmatrix} \xrightarrow{RREF} \begin{pmatrix} 1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

a) Find a basis $\mathcal{B}$ for Nul(A).

b) Is $x = \begin{pmatrix} 2 \\ 3 \\ -10 \\ 1 \end{pmatrix}$ in Nul(A)? If so, find $[x]_{\mathcal{B}}$. If not, justify why x is not in Nul(A).

c) Is $\begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix}$ in Col(A)? You do not need to justify your answer.

Extra space for work on problem 4

Problem 5.

[8 points]

Parts (a), (b), and (c) are unrelated.
You do not need to show any work for parts (a) and (b).

a) I. Is the set $\left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \end{pmatrix} \right\}$ linearly independent? YES NO

II. If A is a 3×3 matrix, is it possible that $\mathrm{Col}(A) = \mathrm{Nul}(A)$?
YES NO

b) Give a specific example of a subspace of $\mathbf{R}^3$ that contains $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$.

(You may express this subspace any way you like, as long as you are clear.)

c) Write a 2×3 matrix A and a 3×2 matrix B so that AB is the standard matrix for the transformation of *clockwise* rotation by $90°$ in $\mathbf{R}^2$.
Compute AB to demonstrate that your answer is correct.

Extra space for work on problem 5