MATH 1553, SPRING 2022
MIDTERM 2

Name		**GT ID**	

Circle your lecture below.

Jankowski, lecture A (8:25-9:15 AM) Jankowski, lecture D (9:30-10:20 AM)

Yu, lecture G (12:30-1:20 PM)

Leykin, lecture I (2:00-2:50 PM) Leykin, lecture M (3:30-4:20 PM)

Please **read all instructions** carefully before beginning.

- Write your initials at the top of each page.
- The maximum score on this exam is 70 points, and you have 75 minutes to complete this exam. Each problem is worth 10 points.
- There are no calculators or aids of any kind (notes, text, etc.) allowed.
- As always, RREF means "reduced row echelon form."
- Show your work, unless instructed otherwise. A correct answer without appropriate work will receive little or no credit!
- We will hand out loose scrap paper, but it **will not be graded** under any circumstances. All answers and all work must be written on the exam itself, with no exceptions.
- This exam is double-sided. You should have more than enough space to do every problem on the exam, but if you need extra space, you may use the *back side of the very last page of the exam*. If you do this, you must clearly indicate it.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

Please read and sign the following statement.

I, the undersigned, hereby affirm that I will not share the contents of this exam with anyone. Furthermore, I have not received inappropriate assistance in the midst of nor prior to taking this exam. I will not discuss this exam with anyone in any form until after 9:15 PM on Wednesday, March 9.

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Problem 1.

For each statement, answer TRUE or FALSE. If the statement is *ever* false, circle FALSE. You do not need to show any work, and there is no partial credit. Each question is worth 2 points.

a) Suppose A is a matrix and $Ax = b$ has exactly one solution for some vector b. Then the columns of A are linearly independent.

TRUE FALSE

b) Let V be the subspace of $\mathbf{R}^3$ consisting of all vectors $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ that satisfy $3x - 2y + z = 0$.

Then $\left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \right\}$ is a basis for V.

TRUE FALSE

c) Suppose A is a 20×25 matrix and $\mathrm{rank}(A) = 5$. Then the dimension of the null space of A is 15.

TRUE FALSE

d) Suppose A is a matrix with column vectors v_1, v_2, v_3 that satisfy $v_1 + v_2 = v_3$. Then the matrix transformation $T(x) = Ax$ cannot be one-to-one.

TRUE FALSE

e) If A is a 5×3 matrix and B is a 3×5 matrix, then the matrix transformation given by $T(x) = ABx$ cannot be onto.

TRUE FALSE

Problem 2.

There is no work required and no partial credit on this page.

a) (3 points) Answer the three questions below.

(I). Let $S = \left\{ \begin{pmatrix} a \\ b \end{pmatrix} \text{ in } \mathbf{R}^2 \ : \ ab = 0 \right\}$. Is S a subspace of $\mathbf{R}^2$? YES NO

(II). Let $V = \left\{ \begin{pmatrix} a \\ b \end{pmatrix} \text{ in } \mathbf{R}^2 \ : a = b \right\}$. Is V a subspace of $\mathbf{R}^2$? YES NO

(III). Let W be the set of all vectors in $\mathbf{R}^3$ of the form $\begin{pmatrix} a \\ b \\ a-b \end{pmatrix}$, where a and b are real numbers. Is W a subspace of $\mathbf{R}^3$? YES NO

b) (2 points) Suppose $u = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $v = \begin{pmatrix} 6 \\ c \end{pmatrix}$.

What values of c will make the set $\{u, v\}$ linearly **dependent**? Clearly circle your answer below.

(i) $c = 0$ only

(ii) $c = 2$ only

(iii) $c = 3$ only

(iv) $c = 9$ only

(v) All c except $c = 2$

(vi) All c except $c = 3$

(vii) All c except $c = 9$

c) (2 points) Suppose $\begin{pmatrix} 1 & -3 & 1 \end{pmatrix}$ is one row in matrix A. Which one of the following vectors **cannot** be in the null space of A? Clearly circle your answer below.

(i) $\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ (ii) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ (iii) $\begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix}$ (iv) $\begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$

d) (3 points)

Suppose A is a 3×4 matrix and the parametric vector form of the solutions to $Ax = 0$ is

$$x_3 \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}.$$

Which of the following statements must be true? Circle all that apply.

(i) $Ax = b$ is consistent for every b in $\mathbf{R}^3$.

(ii) If b is a vector in the column space of A, then the solution set of $Ax = b$ is a line in $\mathbf{R}^4$.

(iii) The first three columns of A form a basis for Col(A).

Problem 3.

Parts (a), (b), and (c) are unrelated. You do not need to show your work or justify your answers for (a) and (b), but show your work for part (c).

a) (4 points)

Let $A = \begin{pmatrix} 1 & -1 \\ 0 & 2 \\ 0 & 3 \end{pmatrix}$, and let T be its associated matrix transformation $T(x) = Ax$.

Answer the following questions. Clearly circle your answer in parts (i), (ii), and (iii).

(i) What is the domain of T? $\quad \mathbf{R} \quad \mathbf{R}^2 \quad$ a plane in $\mathbf{R}^3 \quad \mathbf{R}^3$

(ii) What is the codomain of T? $\quad \mathbf{R} \quad \mathbf{R}^2 \quad$ a plane in $\mathbf{R}^3 \quad \mathbf{R}^3$

(iii) Are there vectors x and y so that $x \neq y$ but $T(x) = T(y)$? $\quad$ YES $\quad$ NO

(iv) Write one vector v in the codomain of T that is not in the range of T.

b) (2 points) Suppose that $T : \mathbf{R}^3 \to \mathbf{R}^4$ is a one-to-one linear transformation and $T\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 0 \end{pmatrix}$. Which one of the following matrices A could possibly be the standard matrix for T? Clearly circle your answer.

(i) $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$ $\quad$ (ii) $A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$ $\quad$ (iii) $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \end{pmatrix}$

(iv) $A = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \\ 0 & 0 & 0 \end{pmatrix}$ $\quad$ (v) There is no such linear transformation T

c) (4 points) Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation that first reflects vectors across the y-axis, then reflects across the line $y = x$. Find the standard matrix A for T.

Problem 4.

You do not need to show your work on this problem, and there is no partial credit. Part (a) is 4 points, (b) is 3 points, and (c) is 3 points.

a) For each matrix A below, let T be the matrix transformation $T(x) = Ax$. If T is one-to-one, circle "one-to-one." If T is onto, circle "onto." If T is one-to-one and onto, circle both. If T is neither one-to-one nor onto, do not circle anything.

(I) $A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ one-to-one onto

(II) $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ one-to-one onto

(III) $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$ one-to-one onto

(IV) $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ one-to-one onto

b) Let A and B be invertible $n \times n$ matrices. Which statements below must be true? Clearly circle all that apply.

(i) $(AB)^{-1} = A^{-1}B^{-1}$.

(ii) $(A+B)(A+B) = A^2 + 2AB + B^2$.

(iii) A and B have the same RREF.

c) Match each of the three matrices below with its corresponding transformation (choosing from (i) through (vii)) by clearly writing that roman numeral in the space provided. We use the usual notation of (x, y) to denote points in $\mathbf{R}^2$.

$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ is the standard matrix for ________

$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ is the standard matrix for ________

$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ is the standard matrix for ________

(i) Projection onto the x-axis in $\mathbf{R}^2$.
(ii) Reflection across the y-axis in $\mathbf{R}^2$.
(iii) Reflection across the line $y = x$ in $\mathbf{R}^2$.
(iv) Reflection across the line $y = -x$ in $\mathbf{R}^2$.
(v) Rotation counterclockwise by $\pi/4$ radians in $\mathbf{R}^2$.
(vi) Rotation clockwise by $\pi/4$ radians in $\mathbf{R}^2$.
(vii) Projection onto the y-axis in $\mathbf{R}^2$.

Problem 5.

Show your work on parts (b) and (c) of this problem.
Consider the matrix A and its reduced row echelon form given below.

$$A = \begin{pmatrix} 1 & -2 & 0 & 1 \\ 1 & -2 & 2 & 3 \\ -1 & 2 & 3 & 2 \end{pmatrix} \xrightarrow{RREF} \begin{pmatrix} 1 & -2 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

a) (2 points) Write a basis for Col A. There is no work required on this part.

b) (4 points) Find a basis for Nul A.

c) (2 points) Is $\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ in the column space of A? Briefly justify your answer.

d) (2 points) Write one nonzero vector in the null space of A. There is no work required and no partial credit for this part.

Problem 6.

Show your work except on parts (c) and (d) of this problem.

Let $T : \mathbf{R}^2 \to \mathbf{R}^3$ be the transformation given by $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 0 \\ x-y \end{pmatrix}$, and let $U : \mathbf{R}^2 \to \mathbf{R}^2$ be the transformation of rotation counterclockwise by 90 degrees.

a) (2 points) Write the standard matrix A for T.

b) (2 points) Write the standard matrix B for U.

c) (1 point) Is T one-to-one? YES NO

d) (1 point) Circle the composition that makes sense: $T \circ U$ $U \circ T$.

e) (4 points) Write the standard matrix for the composition you chose above.

Problem 7.

Free response. Show your work!

a) (2 points) Find A^{-1} if $A = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$.

b) (4 points) Write a matrix A so that the column space of A is the span of $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ and the null space of A is the line $y = -x$ in $\mathbf{R}^2$.

c) (4 points) Let $A = \begin{pmatrix} 1 & 0 & -2 \\ 1 & 1 & 0 \\ 0 & 2 & k \end{pmatrix}$. Find all values of k so that A is invertible.

This page is reserved ONLY for work that did not fit elsewhere on the exam.

If you use this page, please clearly indicate (on the problem's page and here) which problems you are doing.