# MATH 1553, JANKOWSKI
# MIDTERM 2, SPRING 2019

| Name | | Section | |
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Please **read all instructions** carefully before beginning.

- Write your name on the front of each page (not just the cover page!).
- The maximum score on this exam is 50 points, and you have 50 minutes to complete this exam.
- There are no calculators or aids of any kind (notes, text, etc.) allowed.
- As always, RREF means “reduced row echelon form."
- Show your work, unless instructed otherwise. A correct answer without appropriate work will receive little or no credit! If you cannot fit your work on the front side of the page, use the back side of the page and indicate that you are using the back side.
- We will hand out loose scrap paper, but it **will not be graded** under any circumstances. All work must be written on the exam itself.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

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## Problem 1. [10 points]

True or false. Circle **T** if the statement is *always* true.
Otherwise, circle **F**. You do not need to show work or justify your answer.

a) **T** **F** If $A$ is a $5 \times 3$ matrix and $B$ is a $4 \times 5$ matrix, then the transformation $T(x) = BAx$ has domain $\mathbf{R}^3$ and codomain $\mathbf{R}^4$.

b) **T** **F** There is a $4 \times 7$ matrix $A$ that satisfies $\dim(\mathrm{Nul}A) = 1$.

c) **T** **F** Suppose $A$ is an $n \times n$ matrix and the matrix transformation $T$ given by $T(x) = Ax$ is onto. Then $T$ must also be one-to-one.

d) **T** **F** Suppose $A$ is an $n \times n$ matrix and $Ax = 0$ has only the trivial solution. Then each $b$ in $\mathbf{R}^n$ can be written as a linear combination of the columns of $A$.

e) **T** **F** Suppose $v_1, v_2, v_3, v_4$ are vectors in $\mathbf{R}^5$, so that $\mathrm{Span}\{v_1, v_2\}$ has dimension 2 and $\mathrm{Span}\{v_3, v_4\}$ has dimension 2. Then $\mathrm{Span}\{v_1, v_2, v_3, v_4\}$ has dimension 4.

**Extra space for scratch work on problem 1**

## Problem 2.

[9 points]

You do not need to show your work in parts (a)-(c), but show your work in (d).

**a)** Let $V = \text{Span}\left\{ \begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\\0\end{pmatrix}, \begin{pmatrix}1\\2\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\0\\0\end{pmatrix} \right\}$.

Fill in the blank: $\dim(V) =$ ____________.

**b)** Suppose $A$ is a $3 \times 5$ matrix, and the range of the transformation $T(x) = Ax$ is a plane. Fill in the blank:

Nul($A$) is a ________-dimensional subspace of $\mathbf{R}^{\square}$.

**c)** Suppose that $T : \mathbf{R}^n \to \mathbf{R}^m$ is a linear transformation with standard matrix $A$. Which of the following conditions *guarantee* that $T$ is one-to-one? Circle all that apply.

(i) For each $x$ in $\mathbf{R}^n$, there is a unique $y$ in $\mathbf{R}^m$ so that $T(x) = y$.

(ii) For each $y$ in $\mathbf{R}^m$, the matrix equation $Ax = y$ is consistent.

(iii) The columns of $A$ are linearly independent.

**d)** (3 points) Suppose $A$ is a $2 \times 2$ matrix and $A^{-1} = \begin{pmatrix}5 & 4\\2 & 2\end{pmatrix}$. Find $A$.

**Extra space for work on problem 2**

## Problem 3.

[11 points]

Let $T : \mathbf{R}^3 \to \mathbf{R}^2$ be the transformation $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} y - z \\ x + 2y \end{pmatrix}$, and let $U : \mathbf{R}^2 \to \mathbf{R}^2$ be the transformation of rotation counterclockwise by 90°.

**a)** Write the standard matrix $A$ for $T$.

**b)** Write the standard matrix $B$ for $U$.

**c)** Is $T$ onto? YES NO

**d)** Is $U$ invertible? YES NO

**e)** Circle the composition that makes sense: $T \circ U$ $U \circ T$

**f)** Write the standard matrix for the composition you chose in part (e).

**Extra space for work on problem 3**

## Problem 4.

[11 points]

Parts (a) and (b) are unrelated.

**a)** Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation satisfying $T\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\2\end{pmatrix}$ and $T\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}-1\\1\end{pmatrix}$. Find the standard matrix $A$ for $T$.

**b)** Rollo Tomasi has put the matrix $A$ below in its reduced row echelon form:

$$A = \begin{pmatrix} 2 & 6 & -14 & 7 \\ 3 & 9 & -21 & 10 \\ 4 & 12 & -28 & 12 \end{pmatrix} \xrightarrow{RREF} \begin{pmatrix} 1 & 3 & -7 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

(i) Write a basis for Col $A$ (you don't need to justify your answer).

(ii) Write a new basis for Col$A$, so that no vector in your new basis is a scalar multiple of any of the vectors in the basis you wrote in part (i). Clearly show how you obtain this new basis.

(iii) Find one nonzero vector $x$ that satisfies $Ax = 0$.

**Extra space for work on problem 4**

## Problem 5.

[9 points]

Parts (a) and (b) are unrelated. You don't need to justify your answers in part (a).

**a)** Consider the set $V = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \text{ in } \mathbf{R}^2 \;\middle|\; xy \geq 0 \right\}$.

(i) Does $V$ contain the zero vector? YES NO

(ii) Is $V$ closed under addition? YES NO

(iii) Is $V$ closed under scalar multiplication? YES NO

**b)** Consider the subspace $W$ of $\mathbf{R}^3$ given by

$$W = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ in } \mathbf{R}^3 \;\middle|\; x - 5y + 6z = 0 \right\}.$$

(i) Find a basis for $W$.

(ii) Is there a matrix $A$ so that $\mathrm{Col}(A) = W$? If your answer is yes, write such a matrix $A$. If your answer is no, justify why there is no such matrix $A$.

**Extra space for work on problem 5**