MATH 1553, JANKOWSKI (A1-A6)
MIDTERM 2, FALL 2018

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Write your section number here: ____________

Please **read all instructions** carefully before beginning.

- Please leave your GT ID card on your desk until your TA matches your exam.
- The maximum score on this exam is 50 points, and you have 50 minutes to complete this exam.
- There are no calculators or aids of any kind (notes, text, etc.) allowed.
- As always, RREF means "reduced row echelon form."
- Show your work, unless instructed otherwise. A correct answer without appropriate work will receive little or no credit! If you cannot fit your work on the front side of the page, use the back side of the page and indicate that you are using the back side.
- We will hand out loose scrap paper, but it **will not be graded** under any circumstances. All work must be written on the exam itself.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

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Problem 1.

[2 points each]

Answer true if the statement is *always* true. Otherwise, answer false. You do not need to justify your answer.

a) **T** **F** Suppose v_1, v_2, v_3 are vectors in $\mathbf{R}^4$. If $\{v_1, v_2\}$ is linearly independent and v_3 is not in $\text{Span}\{v_1, v_2\}$, then $\{v_1, v_2, v_3\}$ must be linearly independent.

b) **T** **F** If A is a matrix with more rows than columns, then the matrix transformation $T(x) = Ax$ cannot be onto.

c) **T** **F** Suppose that V is a 2-dimensional subspace of $\mathbf{R}^3$ and that $\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ are in V. Then $\left\{ \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \right\}$ is a basis for V.

d) **T** **F** If A is a 3×3 matrix, then Col A must contain the vector $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

e) **T** **F** Suppose $T : \mathbf{R}^n \to \mathbf{R}^m$ is a linear transformation with standard matrix A. If T is not one-to-one, then $Ax = 0$ must have infinitely many solutions.

Scrap paper for problem 1

Problem 2.

[11 points]

Short answer. In this problem, you don't need to justify your answers.

a) Which of the following are subspaces of $\mathbf{R}^3$? Circle all that apply.

(i) The set of solutions to the equation $\begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & 4 \\ 1 & 0 & 3 \end{pmatrix} x = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

(ii) $W = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ in } \mathbf{R}^3 \;\middle|\; 2x - y + z = 0 \right\}$.

(iii) The set $\left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\}$.

b) Let A be a 4×6 matrix, and let T be the matrix transformation $T(x) = Ax$. Which of the following are **possible**? Circle all that apply.

(i) Nul A is a line through the origin.

(ii) For every b in $\mathbf{R}^4$, the equation $Ax = b$ is consistent.

(iii) $\dim(\text{Col } A) = 6$.

(iv) For some b in $\mathbf{R}^4$, the equation $T(x) = b$ has a unique solution.

(v) For every b in $\mathbf{R}^4$, the equation $T(x) = b$ has at most one solution.

c) Fill in the blank: The dimension of $\text{Span}\left\{ \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$ is ______.

d) Write a 2×2 matrix A that is not the identity matrix and that satisfies $A^2 = I$.

Space for extra work on problem 2

Problem 3.

[8 points]

Burt Macklin has row-reduced the matrix A below into its RREF.

$$A = \begin{pmatrix} 1 & -2 & 3 & 3 \\ 2 & -4 & 6 & 2 \\ -1 & 2 & -3 & 1 \end{pmatrix} \xrightarrow{RREF} \begin{pmatrix} 1 & -2 & 3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

a) Write a basis for Col A. (no justification required for this part)

b) Find a basis for Nul A.

c) Let T be the matrix transformation $T(x) = Ax$. Is there a vector in the codomain of T which is not in the range of T? Briefly justify your answer.

Space for extra work on problem 3

Problem 4.

[11 points]

Let $T : \mathbf{R}^3 \to \mathbf{R}^2$ be the transformation

$$T(x, y, z) = (x - 3y + z,\ z - 2x),$$

and let $U : \mathbf{R}^2 \to \mathbf{R}^2$ be the transformation of *clockwise* rotation by 45°.

a) Write the standard matrix A for T and the standard matrix B for U. Simplify your answers (do not leave anything in terms of sine or cosine).

b) Circle all transformations that are one-to-one. T U

c) Circle the composition that makes sense: $U \circ T$ $T \circ U$

d) Compute the standard matrix of the composition you circled in (c).

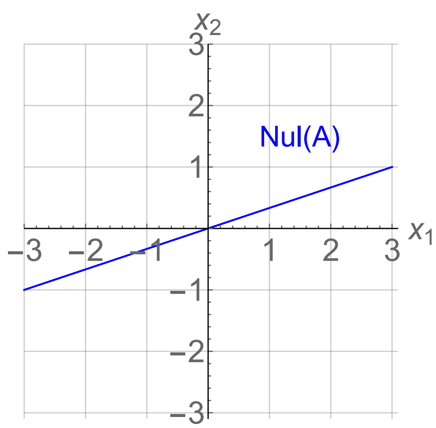
Space for extra work on problem 4

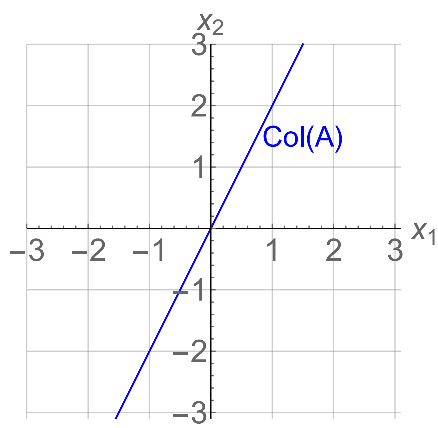
Problem 5. [10 points]

Parts (a) and (b) are unrelated.

a) Find all values of h so that $\left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}, \begin{pmatrix} 4 \\ h \\ 5 \end{pmatrix} \right\}$ is linearly independent.

b) Find a matrix A whose null space and column space are drawn below:





Space for extra work on problem 5