MATH 1553, C. JANKOWSKI
MIDTERM 2

Name		Section	

Please **read all instructions** carefully before beginning.

- You have 50 minutes to complete this exam.
- There are no aids of any kind (calculators, notes, text, etc.) allowed.
- Please show your work unless specified otherwise. A correct answer without appropriate work may be given little or no credit.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

Scoring Page

Please do not write on this page.

1	2	3	4	5	Total

Problem 1.

[Parts a) through f) are worth 2 points each]

a) Complete the following definition (be mathematically precise!):
A set of vectors $\{v_1, v_2, \ldots, v_p\}$ in $\mathbf{R}^n$ is *linearly independent* if...

b) Let $A = \begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix}$. If A is invertible, find A^{-1}. If A is not invertible, justify why.

The remaining problems are true or false. Answer true if the statement is *always* true. Otherwise, answer false. You do not need to justify your answer.

c) **T** **F** If A is an $n \times n$ matrix and the columns of A span $\mathbf{R}^n$, then $Ax = 0$ has only the trivial solution.

d) **T** **F** If A is a 6×7 matrix and the null space of A has dimension 4, then the column space of A is a 2-plane.

e) **T** **F** If A is an $n \times n$ matrix and $Ax = b$ has exactly one solution for some b in $\mathbf{R}^n$, then A is invertible.

f) **T** **F** If A is an $m \times n$ matrix and $m > n$, then the linear transformation $T(x) = Ax$ cannot be one-to-one.

Problem 2.

[10 points]

Parts (a), (b), and (c) are unrelated.

a) Let $V = \text{Span}\left\{\begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}2\\2\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}\right\}$.

Fill in the blank: the dimension of V is ________.

b) Let $W = \left\{\begin{pmatrix}x\\y\\z\end{pmatrix} \text{ in } \mathbf{R}^3 \;\middle|\; x - y - z = 0\right\}$.

Is W a subspace of $\mathbf{R}^3$? (no justification required)

YES NO

c) The famous philologist is obsessed with the set of vectors

$$\left\{\begin{pmatrix}-1\\3\\-1\end{pmatrix}, \begin{pmatrix}1\\1\\-1\end{pmatrix}, \begin{pmatrix}1\\h\\-7\end{pmatrix}\right\}$$

where h is some real number.

Find all values of h that make the set linearly dependent.

Problem 3.

[11 points]

Let $T : \mathbf{R}^3 \to \mathbf{R}^2$ be the linear transformation given by

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_3 - x_2 \\ 2x_1 \end{pmatrix},$$

and let $U : \mathbf{R}^2 \to \mathbf{R}^2$ be reflection about the line $y = x$.

a) Write the standard matrix A for T and the standard matrix B for U.

b) Is U one-to-one? Briefly justify your answer.

c) Find the standard matrix for $U \circ T$.

d) Is the transformation $U \circ T$ onto? Briefly justify your answer.

Problem 4.

[10 points]

Consider the following matrix A and its reduced row echelon form:

$$\begin{pmatrix} 1 & -2 & 4 \\ 0 & 0 & 1 \\ 1 & -2 & 3 \\ -2 & 4 & -8 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & -2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

a) Find a basis for Nul A.

b) Find a basis $\mathcal{B}$ for Col A.

c) Let $x = \begin{pmatrix} -2 \\ -1 \\ -1 \\ 4 \end{pmatrix}$. Is x in Col A?

If your answer is no, justify why x is not in Col A.
If your answer is yes, find $[x]_{\mathcal{B}}$.

Problem 5.

[7 points]

Parts (a) and (b) are unrelated.

a) Suppose that a linear transformation $T : \mathbf{R}^2 \to \mathbf{R}^2$ satisfies $T\begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $T\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$. Find $T\begin{pmatrix} 4 \\ -1 \end{pmatrix}$.

b) Write a single matrix A that satisfies both of the following two properties:

- Col A is a subspace of $\mathbf{R}^4$, and
- Nul A is the line $y = 10x$ in $\mathbf{R}^2$.

[Scratch work]