Math 1553 Exam 2, Fall 2025, Version A

Name		**GT ID**	

Circle your instructor and lecture below. Be sure to circle the correct choice!

Jankowski (A, 8:25 AM) Kim (B, 8:00 AM) Kim (C, 9:00 AM)

Callis (D, 10:00 AM) Short (E, 9:30 AM) Shi (F, 11:00 AM)

Short (H, 12:30 PM) He (I, 2:00 PM) Stokolosa (L, 3:30 PM)

Van Why (M, 3:30 PM) Yap (N, 5:00 PM)

Please read the following instructions carefully.

- Write your initials at the top of each page. The maximum score on this exam is 70 points, and you have 75 minutes to complete it. Each problem is worth 10 points.
- Calculators and cell phones are not allowed. Aids of any kind (notes, text, etc.) are not allowed. If you use pen, you must use black ink.
- As always, RREF means "reduced row echelon form." The "zero vector" in $\mathbf{R}^n$ is the vector in $\mathbf{R}^n$ whose entries are all zero.
- On free response problems, show your work, unless instructed otherwise. A correct answer without appropriate work may receive little or no credit!
- We will hand out loose scrap paper, but it **will not be graded** under any circumstances. All answers and work must be written on the exam itself, with no exceptions.
- This exam is double-sided. You should have enough space to do every problem on the exam, but if you need extra space, you may use the *back side of the very last page of the exam.* If you do this, you must clearly indicate it.
- You may cite any theorem proved in class or in the sections we covered in the text.
- For questions with bubbles, either fill in the bubble completely or leave it blank. **Do not** mark any bubble with "X" or "/" or any such intermediate marking. Anything other than a blank or filled bubble may result in a 0 on the problem, and regrade requests may be rejected without consideration.

I, the undersigned, hereby affirm that I will not share the contents of this exam with anyone. Furthermore, I have not received inappropriate assistance in the midst of nor prior to taking this exam. I will not discuss this exam with anyone in any form until after 7:45 PM on Wednesday, October 15.

____

This page was intentionally left blank.

1. TRUE or FALSE. Clearly fill in the bubble for your answer. If the statement is *ever* false, fill in the bubble for False. You do not need to show any work, and there is no partial credit. Each question is worth 2 points.

   (a) Suppose A is an $m \times n$ matrix. If A has a row of zeros, then the columns of A must be linearly dependent.

   ○ True

   ○ False

   (b) Let V be the subspace of $\mathbf{R}^3$ consisting of all vectors $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ satisfying the equation

   $$5x_1 + x_2 + x_3 = 0.$$

   Then $\left\{ \begin{pmatrix} 1 \\ -4 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \right\}$ is a basis for V.

   ○ True

   ○ False

   (c) If $T : \mathbf{R}^n \to \mathbf{R}^m$ is a linear transformation and $n < m$, then T cannot be one-to-one.

   ○ True

   ○ False

   (d) If A is an $m \times n$ matrix, then its null space must be a subspace of $\mathbf{R}^n$.

   ○ True

   ○ False

   (e) Suppose A is a 4×2 matrix and B is a 2×8 matrix, and let T be the linear transformation given by $T(x) = ABx$. Then the domain of T is $\mathbf{R}^8$ and the codomain of T is $\mathbf{R}^4$.

   ○ True

   ○ False

2. On this page, you do not need to show work, and only your answers are graded. Parts (a) through (d) are unrelated.

   (a) (2 points) Find all real values of c (if there are any) so that the set below is linearly independent.

$$\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 5 \end{pmatrix}, \begin{pmatrix} 1 \\ c \\ 10 \end{pmatrix} \right\}$$

   ○ $c = 0$ only ○ $c = 1$ only ○ $c = 2$ only ○ $c = 5$ only

   ○ $c = -1$ only ○ $c = -2$ only ○ $c = -10$ only

   ○ All c except -1 ○ All c except -2 ○ All c except 2

   ○ The set cannot be linearly independent ○ none of these

   (b) (2 points) Suppose v_1, v_2, and v_3 are vectors in $\mathbf{R}^n$, and let A be the matrix whose columns are v_1, v_2, and v_3. Which **one** of the following statements guarantees that $\{v_1, v_2, v_3\}$ is linearly independent?

   ○ No column of A is a scalar multiple of any other column of A.

   ○ The zero vector is a solution to $Ax = 0$.

   ○ For every vector b in Span$\{v_1, v_2, v_3\}$, the equation $x_1v_1 + x_2v_2 + x_3v_3 = b$ has exactly one solution.

   ○ A has three rows.

   (c) (3 points) Let V be the set of vectors $\begin{pmatrix} x \\ y \end{pmatrix}$ in $\mathbf{R}^2$ that satisfy $y \geq 3x$. Which of the subspace properties does V satisfy? Fill in the bubble for all that apply.

   ○ V contains the zero vector.

   ○ V closed under addition. In other words, if u and v are vectors in V, then $u + v$ must be in V.

   ○ V closed under scalar multiplication. In other words, if u is a vector in V and c is a scalar, then cu must be in V.

   (d) (3 points) Let W be a 3-dimensional subspace of $\mathbf{R}^5$. Which of the following statements must be true? Fill in the bubble for all that apply.

   ○ Every basis of W has exactly 3 vectors.

   ○ Every vector in $\mathbf{R}^5$ is a linear combination of vectors in W.

   ○ If $\{a, b, c\}$ is a linearly independent set of vectors in W, then $\{a, b, c\}$ must be a basis for W.

3. On this page, you do not need to show work, and only your answers are graded. Parts (a) through (d) are unrelated.

(a) (2 points) Find the **one** matrix A below that satisfies

$$\mathrm{Col}(A) = \mathrm{Span}\left\{\begin{pmatrix}1\\4\end{pmatrix}\right\} \quad \text{and} \quad \mathrm{Nul}(A) = \mathrm{Span}\left\{\begin{pmatrix}1\\2\end{pmatrix}\right\}.$$

○ $A = \begin{pmatrix}1 & -2\\4 & -8\end{pmatrix}$ ○ $A = \begin{pmatrix}1 & 1\\4 & 2\end{pmatrix}$ ○ $A = \begin{pmatrix}-2 & 1\\-8 & 4\end{pmatrix}$

○ $A = \begin{pmatrix}1 & 1/2\\4 & 2\end{pmatrix}$ ○ $A = \begin{pmatrix}2 & 1\\8 & 4\end{pmatrix}$ ○ $A = \begin{pmatrix}1 & -1/2\\8 & -4\end{pmatrix}$

○ none of these

(b) (3 points) Which of the following transformations are linear? Fill in the bubble for all that apply.

○ $T : \mathbf{R}^2 \to \mathbf{R}^2$ given by $T(x_1, x_2) = (3x_1 - x_2,\ x_1)$

○ $T : \mathbf{R}^2 \to \mathbf{R}$ given by $T(x, y) = x + 1$.

○ $T : \mathbf{R}^2 \to \mathbf{R}^2$ given by $T\begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{pmatrix}\ln(7) & 0\\\sqrt{2} & 1\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}$.

(c) (2 points) Suppose A is a 45×50 matrix and there are 20 free variables in the solution set to the homogeneous equation $Ax = 0$. Which **one** of the following statements is true about the column space of A?

○ $\mathrm{Col}(A)$ is a 20-dimensional subspace of $\mathbf{R}^{50}$.

○ $\mathrm{Col}(A)$ is a 25-dimensional subspace of $\mathbf{R}^{50}$.

○ $\mathrm{Col}(A)$ is a 30-dimensional subspace of $\mathbf{R}^{50}$.

○ $\mathrm{Col}(A)$ is a 25-dimensional subspace of $\mathbf{R}^{45}$.

○ $\mathrm{Col}(A)$ is a 30-dimensional subspace of $\mathbf{R}^{45}$.

(d) (3 points) Which of the following linear transformations are **onto**? Fill in the bubble for all that apply.

○ The transformation $T(x) = Ax$, where $A = \begin{pmatrix}1 & 3 & -4 & 0\\1 & 1 & 0 & 1\\0 & 2 & -4 & -1\end{pmatrix}$.

○ The transformation $T : \mathbf{R}^2 \to \mathbf{R}^2$ given by $T(x_1, x_2) = (x_1 - 2x_2,\ 3x_1 - 6x_2)$

○ The transformation $T(x) = Ax$, where A is a 4×5 matrix whose null space is a line.

4. On this page, you do not need to show work. Only your answers are graded. Parts (a) through (d) are unrelated.

(a) (2 pts) Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation that first reflects each vector $\begin{pmatrix} x \\ y \end{pmatrix}$ across the y-axis, then reflects across the line $y = x$. Solve $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

○ $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ○ $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$ ○ $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ ○ $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ ○ there is no solution

(b) (2 points) Suppose a linear transformation T satisfies

$$T\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad \text{and} \quad T\begin{pmatrix} 5 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}.$$

Compute $T(v)$ for the vector $v = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - 2\begin{pmatrix} 5 \\ -1 \end{pmatrix}$.

○ $\begin{pmatrix} -1 \\ 6 \end{pmatrix}$ ○ $\begin{pmatrix} -9 \\ 4 \end{pmatrix}$ ○ $\begin{pmatrix} 3 & -4 \\ 4 & 2 \end{pmatrix}$ ○ $\begin{pmatrix} 1 \\ 5 \end{pmatrix}$ ○ $\begin{pmatrix} 5 \\ 5 \end{pmatrix}$

○ $\begin{pmatrix} 11 \\ 4 \end{pmatrix}$ ○ $\begin{pmatrix} 1 \\ -6 \end{pmatrix}$ ○ $\begin{pmatrix} 4 \\ 10 \end{pmatrix}$ ○ $\begin{pmatrix} 1 & -10 \\ 2 & 2 \end{pmatrix}$ ○ none of these

(c) (4 points) Let $T : \mathbf{R}^s \to \mathbf{R}^t$ be a linear transformation with standard matrix A. Which of the following statements guarantee that T is one-to-one? Fill in the bubble for all that apply.

○ For every y in $\mathbf{R}^t$, there is at most one x in $\mathbf{R}^s$ so that $T(x) = y$.

○ For every x in $\mathbf{R}^s$, there is at most one y in $\mathbf{R}^t$ so that $T(x) = y$.

○ The range of T is an s-dimensional subspace of $\mathbf{R}^t$.

○ The matrix equation $Ax = 0$ has only the trivial solution.

(d) (2 points) Let $A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \\ 0 & h \end{pmatrix}$. Find all real values of h (if there are any) so that the linear transformation $T(x) = Ax$ is onto.

○ $h = 0$ only ○ $h = 1$ only ○ All h except 1 ○ All h except 5

○ The transformation T cannot be onto, no matter what h is.

5. Free response. Show your work unless otherwise indicated! A correct answer without sufficient work may receive little or no credit.

For this problem, consider the matrix A and its reduced row echelon form given below.

$$A = \begin{pmatrix} 2 & 6 & -8 & 3 \\ 1 & 3 & -4 & 1 \\ -1 & -3 & 4 & 0 \end{pmatrix} \xrightarrow{RREF} \begin{pmatrix} 1 & 3 & -4 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

(a) (2 pts) Write a basis for Col(A). You do not need to show your work on this part.

(b) (3 points) Write a new basis for Col(A), so that no vector in your new basis is a scalar multiple of any of the vectors you wrote in part (a). Clearly show how you obtain this basis.

(c) (4 points) Find a basis for Nul(A).

(d) (1 pt) Find one x so that $Ax = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$. Write your answer here: $x = \begin{pmatrix} \\ \\ \\ \\ \end{pmatrix}$.

You do not need to show your work on this part, and there is no partial credit.

6. Free response. Show your work unless otherwise indicated! A correct answer without appropriate work will receive little or no credit.

Let $T : \mathbf{R}^3 \to \mathbf{R}^2$ be the linear transformation $T(x_1, x_2, x_3) = (x_1 - x_2,\ 4x_1 + 3x_3)$ and let $U : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation that rotates vectors in $\mathbf{R}^2$ by $45°$ counterclockwise.

(a) Find the standard matrix A for T and write it in the space below. Show your work.

$$A = \begin{pmatrix} & \\ & \end{pmatrix}$$

(b) Write the standard matrix B for U. Evaluate any trigonometric functions you write. Do not leave your answer in terms of sine and cosine.

$$B = \begin{pmatrix} & \\ & \end{pmatrix}$$

(c) Which composition makes sense: $T \circ U$ or $U \circ T$? Fill in the correct bubble below. You do not need to show your work on this part.

○ $T \circ U$ ○ $U \circ T$

(d) Compute the standard matrix C for the composition you selected in (c). Put your answer in the space provided below.

$$C = \begin{pmatrix} & \\ & \end{pmatrix}$$

7. Free response. Show your work unless otherwise indicated! A correct answer without sufficient work will receive little or no credit. Parts (a) through (c) are unrelated.

(a) (3 points) Let V be the subspace of $\mathbf{R}^4$ consisting of all vectors of the form

$$\begin{pmatrix} -4a+3b \\ b \\ a+b \\ b \end{pmatrix},$$

where a and b are real. Find a basis for V. Enter it in the space provided below.

(b) (3 points) Let T be the linear transformation $T(x) = Ax$, where A is the matrix $A = \begin{pmatrix} 4 & -1 & 3 \\ 0 & 1 & 2 \end{pmatrix}$. Find $T\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$. Enter your answer in the space below.

(c) (4 points) Suppose A is a 2×2 matrix whose null space is Span $\left\{ \begin{pmatrix} -4 \\ 1 \end{pmatrix} \right\}$. Find a 2×2 matrix B that has at least one nonzero entry and satisfies:

$$AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

Enter your answer in the space below.

$$B = \begin{pmatrix} \quad & \quad \\ \quad & \quad \end{pmatrix}$$

This page is reserved ONLY for work that did not fit elsewhere on the exam.

If you use this page, please clearly indicate (on the problem's page and here) which problems you are doing.