MATH 1553, JANKOWSKI
MIDTERM 1, SPRING 2018, LECTURE A

Name		**GT Email**	@gatech.edu

Write your section number here: ____________

Please **read all instructions** carefully before beginning.

- Please leave your GT ID card on your desk until your TA matches your exam.
- The maximum score on this exam is 50 points.
- You have 50 minutes to complete this exam.
- There are no aids of any kind (notes, text, etc.) allowed.
- Please show your work. If you cannot fit your work on the front side of the page, use the back side of the page as indicated.
- We will hand out loose scrap paper, but it **will not be graded** under any circumstances. All work must be written on the exam itself.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

Problem 1.

[2 points for each part]

These problems are true or false. Circle **T** if the statement is *always* true. Otherwise, answer **F**. You do not need to justify your answer.

a) **T** **F** The augmented matrix $\left(\begin{array}{ccc|c} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \end{array}\right)$ is in reduced row echelon form.

b) **T** **F** The equation $\begin{pmatrix} 1 & -1 & 2 \\ 0 & 4 & 3 \end{pmatrix} x = b$ is consistent for every b in $\mathbf{R}^2$.

c) **T** **F** If the reduced row echelon form of an augmented matrix has a row of zeros, then the system of linear equations corresponding to the augmented matrix has infinitely many solutions.

d) **T** **F** If A is an $m \times n$ matrix and $Ax = b$ has a unique solution for some b in $\mathbf{R}^m$, then $Ax = 0$ has only the trivial solution.

e) **T** **F** If A is a 4×3 matrix and the solution set for $Ax = 0$ is a line, then A has 2 pivots.

Extra space for scratch work on problem 1

Problem 2.

[11 points]

Show your work on parts (a) and (d) (no work necessary for (b) or (c)).

a) Compute $\begin{pmatrix} 2 & -1 & 1 \\ 3 & 0 & -1 \end{pmatrix}\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}$.

b) Write three different vectors v_1, v_2, v_3 in $\mathbf{R}^3$ so that $\mathrm{Span}\{v_1, v_2, v_3\}$ is only a plane.

c) Write an *augmented* 3×3 matrix in reduced row echelon form whose corresponding system of linear equations is *inconsistent*, and which has a pivot in every row.

d) Find all solutions to the vector equation

$$x_1 \begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix} + x_2 \begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 14 \\ -7 \end{pmatrix}.$$

If there are no solutions, justify why the vector equation is inconsistent.

Extra space for work on problem 2

Problem 3.

[10 points]

Fairway Frank is infatuated with the system of linear equations given by

$$3x - 2y = 4$$
$$6x + hy = k,$$

where h and k are some real numbers.

a) Determine all values of h and k (if there are any) so that the system of equations is inconsistent.

b) Determine all values of h and k (if there are any) so that the system of equations has infinitely many solutions.

Extra space for work on problem 3

Problem 4.

[12 points]

Consider the system of equations in x_1, x_2, x_3, and x_4 given below.

$$\begin{aligned} x_1 - x_2 + 2x_3 - 2x_4 &= -1 \\ -x_1 + x_2 - 2x_3 + x_4 &= 2 \\ -4x_1 + 4x_2 - 8x_3 + 6x_4 &= 6. \end{aligned}$$

a) Write this system of linear equations as a vector equation.

b) Write this system of linear equations as a matrix equation $Ax = b$. Specify every entry of A, x, and b.

c) Put an augmented matrix into reduced row echelon form to solve the system of equations. Write your answer in parametric vector form.

Extra space for work on problem 4

Problem 5. [7 points]

Parts (a) and (b) are unrelated.

a) Write a 3×3 matrix A in reduced row echelon form, with the property that the solution set to $Ax = 0$ is $\text{Span}\left\{ \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \right\}$. Briefly justify your answer.

b) Write a vector b in $\mathbf{R}^3$ which is *not* a linear combination of $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$.

You do not need to justify your answer.

Extra space for work on problem 5