MATH 1553, SPRING 2022
MIDTERM 1

Name		GT ID	

Circle your lecture below.

Jankowski, lecture A (8:25-9:15 AM) Jankowski, lecture D (9:30-10:20 AM)

Yu, lecture G (12:30-1:20 PM)

Leykin, lecture I (2:00-2:50 PM) Leykin, lecture M (3:30-4:20 PM)

Please **read all instructions** carefully before beginning.

- Write your initials at the top of each page.
- The maximum score on this exam is 70 points, and you have 75 minutes to complete this exam. Each problem is worth 10 points.
- There are no calculators or aids of any kind (notes, text, etc.) allowed.
- As always, RREF means "reduced row echelon form."
- Show your work, unless instructed otherwise. A correct answer without appropriate work will receive little or no credit!
- We will hand out loose scrap paper, but it **will not be graded** under any circumstances. All answers and all work must be written on the exam itself, with no exceptions.
- This exam is double-sided. You should have more than enough space to do every problem on the exam, but if you need extra space, you may use the *back side of the very last page of the exam*. If you do this, you must clearly indicate it.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

Please read and sign the following statement.

I, the undersigned, hereby affirm that I will not share the contents of this exam with anyone. Furthermore, I have not received inappropriate assistance in the midst of nor prior to taking this exam. I will not discuss this exam with anyone in any form until after 9:15 PM on Wednesday, February 9.

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Problem 1.

True or false. If the statement is *ever* false, circle FALSE. You do not need to show any work, and there is no partial credit. Each question is worth 2 points.

a) If A is a 5×4 matrix A with a pivot in every column, then every vector in $\mathbf{R}^5$ must be in the span of the columns of A.

TRUE FALSE

b) Suppose we are given a consistent system of 4 linear equations in 3 variables and the corresponding augmented matrix has 2 pivots in its reduced row echelon form. Then the set of solutions to the system of linear equations is a plane.

TRUE FALSE

c) The following vector equation is consistent for each b in $\mathbf{R}^2$:

$$x_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} = b.$$

TRUE FALSE

d) If v_1, v_2, and v_3 are vectors in $\mathbf{R}^2$, then we must have $\text{Span}\{v_1, v_2, v_3\} = \mathbf{R}^2$.

TRUE FALSE

e) If A is a 2×3 matrix and b is a vector so that the set of solutions to $Ax = b$ is the span of $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$, then $b = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

TRUE FALSE

Problem 2.

Multiple choice and short answer. You do not need to show work or justify your answers.

a) (3 pts) Answer YES or NO to each of the following questions.

(i) Is the equation $x + \ln(2)y = 4$ a linear equation in x and y?   YES   NO

(ii) Is $(2, 0, 1)$ in the plane $x - y - z = 1$?   YES   NO

(iii) Is it possible for a system of linear equations to have exactly 3 solutions?
YES   NO

b) (2 pts) Write an *inconsistent* system of two linear equations in the three variables x, y, and z.

c) (3 pts) Which of the following matrices are in reduced row echelon form? Circle all that apply.

(i) $\left(\begin{array}{ccc|c} 0 & 0 & 1 & 1 \end{array}\right)$

(ii) $\left(\begin{array}{ccc|c} 1 & -5 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right)$

(iii) $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$

d) (2 pts) Suppose an augmented matrix in RREF has $\left(\begin{array}{ccc|c} 0 & 0 & 1 & 3 \end{array}\right)$ as its bottom row. Which one of the following statements is true about the system of linear equations corresponding to the augmented matrix?

(i) The system must be inconsistent.

(ii) We need more information to determine whether the system is consistent or inconsistent.

(iii) The system must have exactly one solution.

(iv) The system must have infinitely many solutions.

(v) The system must be consistent, but we need more information to determine whether it has a unique solution or infinitely many solutions.

Problem 3.

Multiple choice and short answer. You do not need to show your work or justify your answers.

a) (3 pts) Write a set of three different vectors v_1, v_2, and v_3 in $\mathbf{R}^3$ satisfying both of the following properties:

- The span of any two of the vectors is a plane.
- $\text{Span}\{v_1, v_2, v_3\}$ is also a plane.

b) (2 pts) You are trying to solve this system of equations, but there is a smudge on the page on the coefficient of y in the first equation (the smudge is the mark $\bullet$ below):

$$\begin{aligned} 2x + \bullet y &= 5 \\ 4x - y &= 3 \end{aligned}$$

The answer key says that the solution to the system is $(1,1)$. Which one of the following scenarios is possible?

(i) There must have been a typo, since $(1,1)$ cannot be a solution.

(ii) There is exactly one value for the smudge so that $(1,1)$ is a solution.

(iii) There are multiple possible values for the smudge so that $(1,1)$ is a solution.

c) (5 pts) Suppose A is a matrix and b is a nonzero vector so that the solution set to $Ax = b$ has the following parametric vector form (where x_2 and x_3 are free):

$$\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}.$$

(i) How many columns does A have? Circle your answer below.

2   3   4   not enough information given

(ii) How many rows does A have? Circle your answer below.

2   3   4   not enough information given

(iii) Which of the following vectors must satisfy the homogeneous equation $Ax = 0$? Circle all that apply.

(1) $x = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$ (2) $x = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (3) $x = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

Problem 4.

Short answer and multiple choice. Show your work on part (a).

a) (2 pts) Compute the matrix product $\begin{pmatrix} 1 & 0 & 3 \\ -2 & 3 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$.

b) (2 pts) Suppose A is a matrix and $A\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$. Compute $A\begin{pmatrix} 4 \\ 2 \end{pmatrix}$.

c) (3 pts) For each augmented matrix given below, determine whether the corresponding linear system is consistent. If it is inconsistent, circle "there are no solutions." If the system is consistent, determine whether the solution set is a point, a line, or a plane, and circle your answer.

(i) For $\left(\begin{array}{cc|c} 1 & 0 & 2021 \\ 0 & 0 & 2022 \end{array}\right)$, the solution set is:

a single point   a line   a plane   there are no solutions

(ii) For $\left(\begin{array}{cc|c} 1 & 2 & 1011 \\ 2 & 4 & 2022 \end{array}\right)$, the solution set is:

a single point   a line   a plane   there are no solutions

(iii) For $\left(\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)$, the solution set is:

a single point   a line   a plane   there are no solutions

d) (3 pts) Suppose v_1, v_2, v_3, and b are vectors in $\mathbf{R}^n$. Which of the following statements are true? Circle all that apply.

(i) If b is in $\text{Span}\{v_1, v_2, v_3\}$, then the vector equation $x_1v_1 + x_2v_2 + x_3v_3 = b$ must be consistent.

(ii) If the vector equation $x_1v_1 + x_2v_2 = b$ is consistent, then the vector equation

$$x_1v_1 + x_2v_2 + x_3v_3 = b$$

must also be consistent.

(iii) If b is the zero vector, then the vector equation $x_1v_1 + x_2v_2 + x_3v_3 = b$ must be consistent.

Problem 5.

Free response. Show your work! The two parts of this problem are unrelated.

a) (5 pts) Consider the linear system of equations given by

$$2x - hy = 4$$

$$8x + 16y = k.$$

Find all values of h and k (if there are any) so that the system has infinitely many solutions.

b) (5 pts) Find all values of c so that $\begin{pmatrix} 5 \\ c \\ -2 \end{pmatrix}$ is a linear combination of $\begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$.

Problem 6.

Free response. Show your work on parts (a) and (b).

Consider the following linear system of equations in the variables $x_1,\ x_2,\ x_3,\ x_4$:

$$x_1 - 4x_2 + 0x_3 + 2x_4 = 1$$
$$-x_1 + 4x_2 - x_3 - x_4 = -3$$
$$-2x_1 + 8x_2 - 2x_3 - 2x_4 = -6.$$

a) (4 pts) Write the augmented matrix corresponding to this system, and put the augmented matrix into RREF.

b) (4 pts) The system is consistent. Write the set of solutions to the system of equations in parametric vector form.

c) (2 pts) Write *one* vector which is not the zero vector and which is a solution to the corresponding homogeneous system of equations given below:

$$x_1 - 4x_2 + 0x_3 + 2x_4 = 0$$
$$-x_1 + 4x_2 - x_3 - x_4 = 0$$
$$-2x_1 + 8x_2 - 2x_3 - 2x_4 = 0.$$

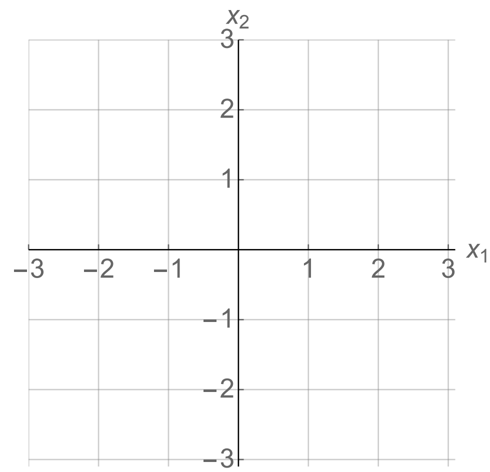
There is no work necessary and no partial credit on part (c).

Problem 7.

Show your work on parts (a) and (b).

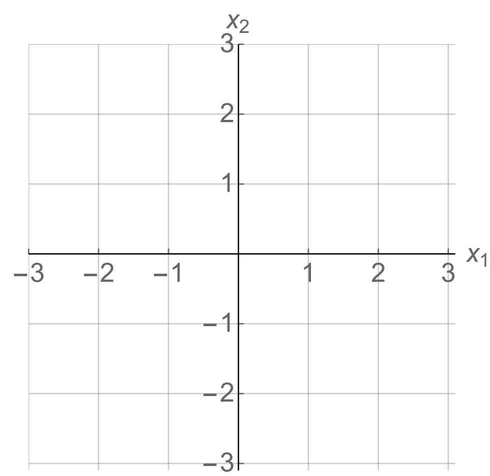
For this entire problem, let $A = \begin{pmatrix} 2 & 1 \\ -8 & -4 \\ 4 & 2 \end{pmatrix}$.

a) (5 pts) Write the solution set to $Ax = 0$ in parametric form, and draw the solution set to $Ax = 0$ on the graph below.



b) (3 pts) For some b, the vector $v = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ is a solution to the equation $Ax = b$.

Do we have enough information to draw the solution set for $Ax = b$? If your answer is no, explain why we do not have enough information. If your answer is yes, draw the solution set to $Ax = b$ on the graph below.



c) (2 pts) You do not need to show work on this part. Answer the following.
The span of the columns of A is a:

(circle one answer) **point** **line** **plane**

in:

(circle one answer) $\mathbf{R}$ $\mathbf{R}^2$ $\mathbf{R}^3$.

This page is reserved ONLY for work that did not fit elsewhere on the exam.

If you use this page, please clearly indicate (on the problem's page and here) which problems you are doing.