MATH 1553, JANKOWSKI
MIDTERM 1, SPRING 2019

Name		**Section**	

Please **read all instructions** carefully before beginning.

- Write your name on the front of each page (not just the cover page!).
- The maximum score on this exam is 50 points, and you have 50 minutes to complete this exam.
- There are no calculators or aids of any kind (notes, text, etc.) allowed.
- As always, RREF means "reduced row echelon form."
- Show your work, unless instructed otherwise. A correct answer without appropriate work will receive little or no credit! If you cannot fit your work on the front side of the page, use the back side of the page and indicate that you are using the back side.
- We will hand out loose scrap paper, but it **will not be graded** under any circumstances. All work must be written on the exam itself.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

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Problem 1. [10 points]

These problems are true or false. Circle **T** if the statement is *always* true. Otherwise, circle **F**. You do not need to justify your answer.

a) **T** **F** The matrix $\left(\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$ is in reduced row echelon form.

b) **T** **F** A system of 4 linear equations in 5 variables can have exactly one solution.

c) **T** **F** The vector equation $x_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} + x_2 \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ is consistent.

d) **T** **F** Suppose A is an 4×3 matrix whose first column is the sum of its second and third columns. Then the equation $Ax = 0$ has infinitely many solutions.

e) **T** **F** If A is an $m \times n$ matrix and $m > n$, then then there is at least one vector b in $\mathbf{R}^m$ which is not in the span of the columns of A.

Extra space for scratch work on problem 1

Problem 2.

[10 points]

Short answer. You do not need to show your work or justify your answer. Parts (a)-(b) are 2 points each. Parts (c) and (d) are worth 3 points each.

a) Complete the following mathematical definition of linear independent (be mathematically precise!):
Let $v_1, v_2 \dots, v_p$ be vectors in $\mathbf{R}^n$. We say $\{v_1, \dots, v_p\}$ is *linearly independent* if...

b) Are there three nonzero vectors $v_1,\ v_2,\ v_3$ in $\mathbf{R}^3$ so that $\text{Span}\{v_1,\ v_2,\ v_3\}$ is a plane but v_3 is not in $\text{Span}\{v_1, v_2\}$? If your answer is yes, write such vectors $v_1,\ v_2,\ v_3$ and label each vector clearly.

c) Write a matrix A with the property that the equation $Ax = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ is consistent.

d) Suppose A is a 2×3 matrix and v is some vector so that the set of solutions to $Ax = v$ has parametric form

$$x_1 = 1 + x_3 \qquad x_2 = 2 - x_3 \qquad x_3 = x_3 \qquad (x_3 \text{ free}).$$

Which of the following must be true? Circle all that apply.

(i) The solution set for $Ax = 0$ is $\text{Span}\left\{ \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right\}$.

(ii) For each b in $\mathbf{R}^2$, the equation $Ax = b$ is consistent.

(iii) v is not the zero vector.

Extra space for work on problem 2

Problem 3. [11 points]

Parts (a) and (b) are unrelated. Part (a) is worth 6 points and (b) is worth 5 points.

a) John Dioguardi cannot stop thinking about the system of equations

$$x - 4y = h$$
$$-3x + ky = 4,$$

where h and k are real numbers.

For what values of h and k (if any) is the system inconsistent?

b) Let $v_1 = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}, v_2 = \begin{pmatrix} 2 \\ 1 \\ -6 \end{pmatrix}, v_3 = \begin{pmatrix} -3 \\ 0 \\ -9 \end{pmatrix}$. Are the vectors v_1, v_2, v_3 linearly independent or linearly dependent? If they are linearly independent, justify why. If they are linearly dependent, write one vector as a linear combination of the other vectors.

Extra space for work on problem 3

Problem 4. [11 points]

Consider the following linear system of equations in the variables $x_1,\ x_2,\ x_3$:

$$\begin{aligned} x_1 - 2x_2 + 2x_3 &= 1 \\ 5x_1 - 10x_2 + 12x_3 &= -3 \\ -3x_1 + 6x_2 - 6x_3 &= -3. \\ 2x_1 - 4x_2 + 5x_3 &= -2. \end{aligned}$$

a) Write the augmented matrix corresponding to this system, and put the augmented matrix into RREF.

b) The system is consistent. Write the set of solutions to the system of equations in parametric vector form.

c) Write *one* specific vector that solves the system of equations.

Extra space for work on problem 4

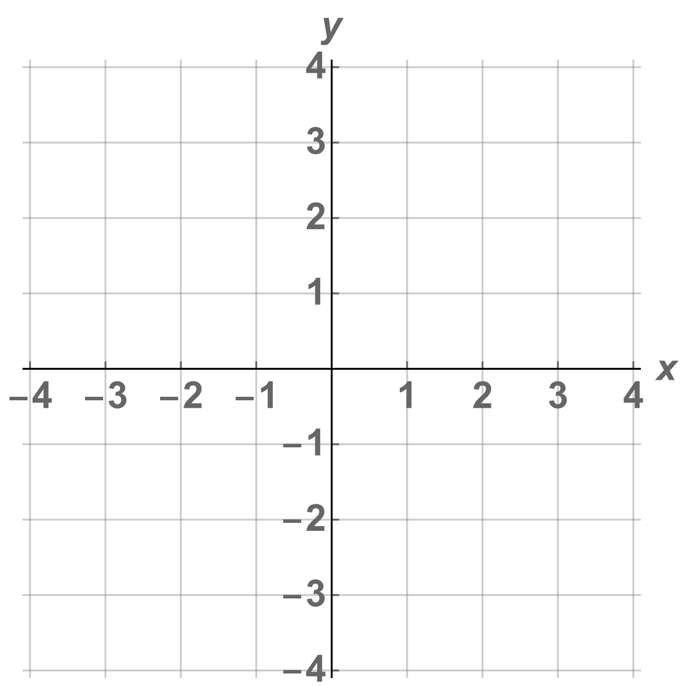
Problem 5.

[8 points]

Parts (a) and (b) are unrelated.

a) Write an augmented matrix in RREF representing a system of three equations in two unknowns, whose solution set is the line $y = 2x$ in $\mathbf{R}^2$.

b) Let $A = \begin{pmatrix} 2 & -4 \\ 1 & -2 \end{pmatrix}$. Draw the span of the columns of A below.



Extra space for work on problem 5