# MATH 1553, JANKOWSKI (A1-A6)
# MIDTERM 1, FALL 2018

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|---|---|---|---|

Write your section number here: ____________

Please **read all instructions** carefully before beginning.

- Please leave your GT ID card on your desk until your TA matches your exam.
- The maximum score on this exam is 50 points, and you have 50 minutes to complete this exam.
- There are no calculators or aids of any kind (notes, text, etc.) allowed.
- As always, RREF means “reduced row echelon form."
- Show your work, unless instructed otherwise. A correct answer without appropriate work will receive little or no credit! If you cannot fit your work on the front side of the page, use the back side of the page and indicate that you are using the back side.
- We will hand out loose scrap paper, but it **will not be graded** under any circumstances. All work must be written on the exam itself.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

## Problem 1.

[10 points]

These problems are true or false. Circle **T** if the statement is *always* true. Otherwise, circle **F**. You do not need to justify your answer.

a) **T** **F** The matrix $\left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right)$ is in reduced row echelon form.

b) **T** **F** If the bottom row of an augmented matrix in RREF is $\left(\begin{array}{ccc|c} 0 & 1 & 2 & 3 \end{array}\right)$, then the corresponding system of equations must be consistent.

c) **T** **F** The vector equation $x\begin{pmatrix} 1 \\ -1 \end{pmatrix} + y\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ has infinitely many solutions.

d) **T** **F** If $A$ is a $3 \times 4$ matrix and $b$ is a vector so that the set of solutions to $Ax = b$ is a line through the origin, then $b = 0$.

e) **T** **F** If $A$ is a $3 \times 3$ matrix and $Ax = 0$ has infinitely many solutions, then $Ax = b$ must be inconsistent for some $b$ in $\mathbf{R}^3$.

**Extra space for scratch work on problem 1**

## Problem 2.

[10 points]

You do not need to show your work in parts (a) through (c).

**a)** [2 points] Suppose we have a consistent system of four linear equations in three variables, and the corresponding augmented matrix has two pivots. The set of solutions to the system is a:

(circle one answer) **point** **line** **plane** **3-plane**

in:

(circle one answer) $\mathbf{R}$ $\mathbf{R}^2$ $\mathbf{R}^3$ $\mathbf{R}^4$.

**b)** [2 pts] Which of the following conditions guarantee that a system of 3 linear equations in 4 variables has *infinitely many solutions*? (circle all that apply)
(1) The reduced row echelon form of the augmented matrix corresponding to the system of equations has a row of zeros.
(2) The rightmost column of the augmented matrix is not a pivot column.

**c)** [2 pts] Write a vector $v$ in $\mathbf{R}^3$ which is NOT a linear combination of $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -5 \\ 0 \end{pmatrix}$.

**d)** [4 pts] Write a matrix $A$, with the property that $Ax = b$ is consistent if and only if $b$ is in the span of $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix}$. Briefly justify your answer.

**Extra space for work on problem 2**

## Problem 3.

[9 points]

Michael Scarn is convinced that the key to his mystery is contained in the system of equations

$$x - ky = 3$$
$$-2x - 4y = h,$$

where $h$ and $k$ are real numbers.

**a)** For what values of $h$ and $k$ (if any) does the system have a unique solution?

**b)** For what values of $h$ and $k$ (if any) does the system have infinitely many solutions?

**Extra space for work on problem 3**

## Problem 4. [10 points]

Consider the following linear system of equations in the variables $x_1,\ x_2,\ x_3,\ x_4$:

$$x_1 - 3x_3 + x_4 = 1$$
$$3x_1 + x_2 - 9x_3 + 5x_4 = 1$$
$$-2x_1 + 2x_2 + 6x_3 + 2x_4 = -6.$$

**a)** Write the augmented matrix corresponding to this system, and put the augmented matrix into RREF.

**b)** Write the set of solutions to the system from part (a) in parametric vector form.

**c)** Write *one* vector which is not the zero vector and solves the corresponding homogeneous system of linear equations:

$$x_1 - 3x_3 + x_4 = 0$$
$$3x_1 + x_2 - 9x_3 + 5x_4 = 0$$
$$-2x_1 + 2x_2 + 6x_3 + 2x_4 = 0.$$

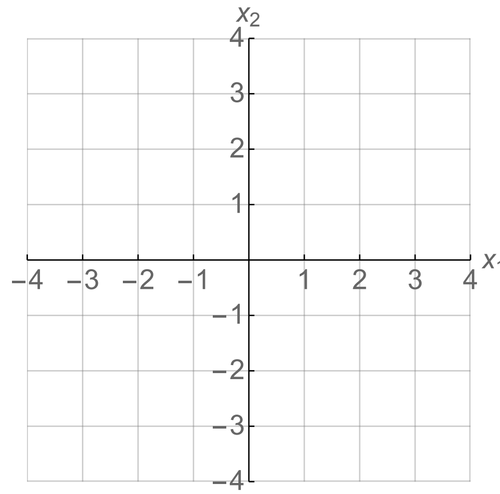
**Extra space for work on problem 4**

## Problem 5.

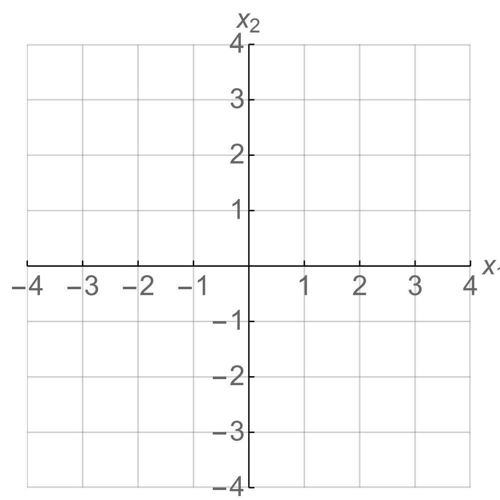
[11 points]

Throughout this problem, let $A = \begin{pmatrix} 2 & -4 \\ -1 & 2 \end{pmatrix}$.

**a)** Draw the solution set for $Ax = 0$ and draw the solution set for $Ax = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ on the same graph below. Clearly label each solution set.




**b)** Draw the span of the columns of $A$ on the graph below.




**c)** Is there a vector $b$ so that $Ax = b$ has a unique solution? Justify your answer.

**Extra space for work on problem 5**