Math 1553 Exam 1, Fall 2025, Version A

Name		**GT ID**	

Circle your instructor and lecture below. Be sure to circle the correct choice!

Jankowski (A, 8:25 AM) Kim (B, 8:00 AM) Kim (C, 9:00 AM)

Callis (D, 10:00 AM) Short (E, 9:30 AM) Shi (F, 11:00 AM)

Short (H, 12:30 PM) He (I, 2:00 PM) Stokolosa (L, 3:30 PM)

Van Why (M, 3:30 PM) Yap (N, 5:00 PM)

Please read the following instructions carefully.

- Write your initials at the top of each page. The maximum score on this exam is 70 points, and you have 75 minutes to complete it. Each problem is worth 10 points.
- Calculators and cell phones are not allowed. Aids of any kind (notes, text, etc.) are not allowed. If you use pen, you must use black ink.
- As always, RREF means "reduced row echelon form." The "zero vector" in $\mathbf{R}^n$ is the vector in $\mathbf{R}^n$ whose entries are all zero.
- On free response problems, show your work, unless instructed otherwise. A correct answer without appropriate work may receive little or no credit!
- We will hand out loose scrap paper, but it **will not be graded** under any circumstances. All answers and work must be written on the exam itself, with no exceptions.
- This exam is double-sided. You should have enough space to do every problem on the exam, but if you need extra space, you may use the *back side of the very last page of the exam.* If you do this, you must clearly indicate it.
- You may cite any theorem proved in class or in the sections we covered in the text.
- For questions with bubbles, either fill in the bubble completely or leave it blank. **Do not** mark any bubble with "X" or "/" or any such intermediate marking. Anything other than a blank or filled bubble may result in a 0 on the problem, and regrade requests may be rejected without consideration.

I, the undersigned, hereby affirm that I will not share the contents of this exam with anyone. Furthermore, I have not received inappropriate assistance in the midst of nor prior to taking this exam. I will not discuss this exam with anyone in any form until after 7:45 PM on Wednesday, September 17.

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1. TRUE or FALSE. Clearly fill in the bubble for your answer. If the statement is *ever* false, fill in the bubble for False. You do not need to show any work, and there is no partial credit. Each question is worth 2 points.

   (a) If an augmented matrix in RREF has a pivot in every row, then its corresponding system of linear equations must be consistent.

   ○ True

   ○ False

   (b) Suppose a linear system of 3 equations in 2 variables is consistent. Then the system must have exactly one solution.

   ○ True

   ○ False

   (c) If the zero vector is a solution to a matrix equation, then the matrix equation must be homogeneous.

   ○ True

   ○ False

   (d) Suppose v_1 and v_2 are vectors in $\mathbf{R}^2$ and that $\text{Span}\{v_1, v_2\}$ is a line. Then there is some vector b in $\mathbf{R}^2$ so that the vector equation $x_1v_1 + x_2v_2 = b$ is inconsistent.

   ○ True

   ○ False

   (e) Let $v_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}$, and $v_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$.

   Then every vector in $\mathbf{R}^3$ can be written as a linear combination of v_1, v_2, and v_3.

   ○ True

   ○ False

2. On this page, you do not need to show work, and only your answers are graded. Parts (a) through (d) are unrelated.

   (a) (3 points) Which of the following equations are **linear** equations in x, y, and z? Clearly fill in the bubble for all that apply.

   ○ $x - yz = 5$

   ○ $3x - 5^{1/3}y + 4z = 1$

   ○ $2x - y - 9z = 0$

   (b) (3 points) Which of the following statements are true? Clearly fill in the bubble for all that apply.

   ○ The matrix $\left(\begin{array}{cccc|c} 0 & 1 & -3 & 0 & -1 \\ 0 & 0 & 0 & 1 & 15 \end{array}\right)$ is in RREF.

   ○ The linear system corresponding to the augmented matrix $\left(\begin{array}{cc|c} 1 & 0 & -3 \\ 0 & 4 & -1 \\ 0 & 8 & -3 \end{array}\right)$ has exactly one solution.

   ○ There is a 3×5 augmented matrix in RREF whose **bottom** row is $\left(\begin{array}{cccc|c} 0 & 1 & 0 & 0 & 0 \end{array}\right)$.

   (c) (2 points) Consider the linear system for the augmented matrix $\left(\begin{array}{ccc|c} 1 & -5 & 0 & 3 \\ -2 & 10 & 1 & 4 \end{array}\right)$. Which **one** of the following is the parametric form for the system's solution set?

   ○ $x_1 = 3 + 5x_2, \quad x_2 = x_2 \ (x_2 \text{ real}), \quad x_3 = 10.$

   ○ $x_1 = 3, \quad x_2 = 0, \quad x_3 = 4.$

   ○ $x_1 = 3 + 5x_2, \quad x_2 = x_2 \ (x_2 \text{ real}), \quad x_3 = 4.$

   ○ $x_1 = 3, \quad x_2 = 0, \quad x_3 = 10.$

   (d) (2 points) Consider the following linear system of equations, where h is some real number:

$$2x + 4y = -4$$
$$6x + hy = 1.$$

   Which **one** of the following statements is true?

   ○ If $h = 12$, then the system has infinitely many solutions.

   ○ If the system is consistent, it must have exactly one solution.

   ○ There is exactly one value of h that makes the system consistent.

   ○ The system must be consistent, regardless of the value of h.

3. On this page, you do not need to show work, and only your answers are graded. Parts (a) through (e) are unrelated.

(a) (2 points) Solve for a and b in the equation $3\begin{pmatrix} a \\ -5 \end{pmatrix} + 2\begin{pmatrix} 1 \\ b \end{pmatrix} = \begin{pmatrix} 8 \\ 1 \end{pmatrix}$.

○ $a = 2$ and $b = -3$ ○ $a = 6$ and $b = -3$ ○ $a = 7$ and $b = 8$

○ $a = 7$ and $b = 6$ ○ $a = 2$ and $b = 8$ ○ none of these

(b) (2 points) Solve $Ax = b$, where $A = \begin{pmatrix} 1 & -2 \\ -2 & 1 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$.

Fill in the bubble for your answer below.

○ $x = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ ○ $x = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$ ○ $x = \begin{pmatrix} -7/5 \\ 6/5 \end{pmatrix}$ ○ $x = \begin{pmatrix} -7 \\ 2 \end{pmatrix}$

(c) (2 points) Consider the vector equation $x_1\begin{pmatrix} 1 \\ 0 \end{pmatrix} + x_2\begin{pmatrix} 0 \\ 1 \end{pmatrix} + x_3\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

Which of the following describes the solution set to the vector equation?

○ a point in $\mathbf{R}^2$ ○ a line in $\mathbf{R}^2$ ○ all of $\mathbf{R}^2$

○ a point in $\mathbf{R}^3$ ○ a line in $\mathbf{R}^3$ ○ a plane in $\mathbf{R}^3$

(d) (2 points) Suppose v_1 and v_2 are vectors in $\mathbf{R}^n$. Which **one** of the following statements **must** be true?

○ Span$\{v_1, v_2\}$ is the same as Span$\{v_1 + v_2,\ -v_1 - v_2\}$.

○ If a vector w is a linear combination of v_1 and v_2, then $-2w$ must also be a linear combination of v_1 and v_2.

○ If $v_1 \neq v_2$, then Span$\{v_1, v_2\}$ is a plane in $\mathbf{R}^n$.

○ If b is in Span$\{v_1, v_2\}$, then the vector equation $x_1v_1 + x_2v_2 = b$ must have exactly one solution.

(e) (2 points) Find all real values of h so that Span$\left\{\begin{pmatrix} -4 \\ 2 \end{pmatrix}, \begin{pmatrix} h \\ 6 \end{pmatrix}\right\} = \mathbf{R}^2$.

○ $h = 0$ only ○ $h = -2$ only ○ $h = -6$ only ○ $h = -12$ only

○ all real h except 0 ○ all real h except -2 ○ all real h except -6

○ all real h except -12 ○ all real h ○ none of these

4. On this page, you do not need to show work. Only your answers are graded. Parts (a) through (d) are unrelated.

(a) (2 points) Compute $\begin{pmatrix} 1 & 2 \\ -3 & 1 \\ 4 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ -5 \end{pmatrix}$.

○ $\begin{pmatrix} -8 \\ -11 \\ 8 \end{pmatrix}$ ○ $\begin{pmatrix} -8 \\ 1 \\ 8 \end{pmatrix}$ ○ $\begin{pmatrix} -8 \\ -11 \\ 0 \end{pmatrix}$ ○ $\begin{pmatrix} -10 \\ -11 \\ 8 \end{pmatrix}$ ○ $\begin{pmatrix} 2 & -10 \\ -6 & -5 \\ 8 & 0 \end{pmatrix}$

(b) (2 points) Suppose a matrix A satisfies $A\begin{pmatrix} 1 \\ -9 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $A\begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

Compute Av for the vector $v = \begin{pmatrix} 1 \\ -9 \end{pmatrix} + 3\begin{pmatrix} 0 \\ 2 \end{pmatrix}$.

○ $Av = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ ○ $Av = \begin{pmatrix} 1 \\ -7 \end{pmatrix}$ ○ $Av = \begin{pmatrix} -3 \\ 3 \end{pmatrix}$ ○ $Av = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

○ $Av = \begin{pmatrix} -9 \\ 2 \end{pmatrix}$ ○ $Av = \begin{pmatrix} 1 & -3 \\ -9 & 3 \end{pmatrix}$ ○ $Av = \begin{pmatrix} 0 & -3 \\ 1 & 3 \end{pmatrix}$

○ $Av = \begin{pmatrix} 1 & 0 \\ -9 & 6 \end{pmatrix}$ ○ not enough info to find Av.

(c) (3 points) Suppose A is a 4×3 matrix whose RREF has two pivots. Answer the following questions by filling in the appropriate bubble in each case.

i. If a vector v is a linear combination of the columns of A, then v is in:
○ $\mathbf{R}^2$ ○ $\mathbf{R}^3$ ○ $\mathbf{R}^4$ ○ not enough info to determine

ii. If a vector w is a solution to $Ax = 0$, then w is in:
○ $\mathbf{R}^2$ ○ $\mathbf{R}^3$ ○ $\mathbf{R}^4$ ○ not enough info to determine

iii. Which one of the following describes the set of solutions to the homogeneous equation $Ax = 0$?
○ a point ○ a line ○ a plane ○ all of $\mathbf{R}^3$ ○ all of $\mathbf{R}^4$

(d) (3 points) Suppose the solution set to some matrix equation $Ax = b$ has parametric vector form $x = \begin{pmatrix} -4 \\ 0 \end{pmatrix} + x_2\begin{pmatrix} 3 \\ 1 \end{pmatrix}$, where x_2 is any real number. Which of the following are solutions to $Ax = 0$? Fill in the bubble for all that apply.

○ $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$ ○ $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ ○ $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ ○ $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

5. Free response. Show your work unless otherwise indicated! A correct answer without sufficient work may receive little or no credit.
For this page of the exam, consider the following linear system of equations in the variables $x_1,\ x_2,\ x_3,\ x_4$:

$$\begin{aligned} x_1 + 3x_3 - 3x_4 &= 2 \\ 2x_1 + x_2 + 6x_3 - 4x_4 &= 6 \\ -2x_1 - 2x_2 - 5x_3 - x_4 &= -7. \end{aligned}$$

(a) (4 points) Write the system in the form of an augmented matrix, and put the augmented matrix in reduced row echelon form.

(b) (4 pts) The system is consistent. Write its solution set in parametric **vector** form.

(c) (2 points) Write two different solutions u and v to the linear system in the space provided below. Only your answer is graded on this part, so please check by hand that your answer is correct, and if it is not correct then check your work above!

$$u = \begin{pmatrix} \\ \\ \\ \\ \end{pmatrix} \qquad v = \begin{pmatrix} \\ \\ \\ \\ \end{pmatrix}$$

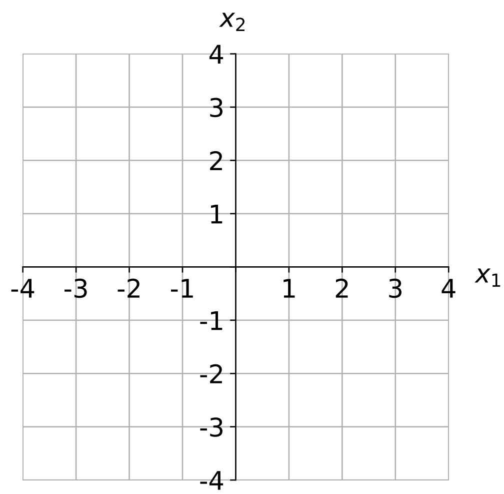
6. Free response. Show your work unless otherwise indicated! A correct answer without appropriate work will receive little or no credit. Parts (a) and (b) are unrelated.

   (a) (5 points) Consider the vector equation

$$x_1 \begin{pmatrix} 1 \\ 5 \end{pmatrix} + x_2 \begin{pmatrix} -2 \\ -10 \end{pmatrix} = \begin{pmatrix} 2 \\ 10 \end{pmatrix}.$$

      i. Find the set of solutions to the vector equation, and write it in parametric form.

      ii. Using your answer from part (i), draw the solution set for the vector equation below very carefully. You do not need to show your work for this part.



   (b) (5 points) Find all values of h and k so that the system below has infinitely many solutions. Clearly mark your answers for h and k.

$$3x - 12y = k$$

$$9x - hy = 10.$$

7. Free response. Show your work unless otherwise indicated! A correct answer without sufficient work will receive little or no credit.

(a) (2 points) Write a matrix A with the property that $Ax = b$ is consistent if and only if b is in Span $\left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$. You do not need to show work on this part.

(b) (3 points) Consider the linear system of system of equations in x_1, x_2, x_3, and x_4 whose augmented matrix is given below:

$$\left(\begin{array}{cccc|c} 1 & -5 & 0 & 1 & 3 \\ 0 & 1 & 1 & -1 & -2 \end{array} \right).$$

Write the set of solutions to the system in parametric form. Indicate which variables (if any) are free.

(c) (5 points) Let $A = \begin{pmatrix} 1 & -3 \\ 3 & 1 \\ 4 & -10 \end{pmatrix}$ and $b = \begin{pmatrix} 2 \\ h \\ 2 \end{pmatrix}$.

Find all real values of h so that the matrix equation $Ax = b$ is **inconsistent**.

This page is reserved ONLY for work that did not fit elsewhere on the exam.

If you use this page, please clearly indicate (on the problem's page and here) which problems you are doing.