Quiz 9, Discrete Math (15 points), Fall 2016

You do not need to simplify your answers. For example, if your answer is $100! \cdot 4 \cdot 7 \cdot 5 - 20$, you can leave your answer in that form.

1. (4 points) How many even numbers between 3000 and 9998 (inclusive) have no repeated digits?
   To be clear: by "repeated digit" we mean a digit that appears at least twice in the number.

   Solution: The final digit is 0, 2, 4, 6, or 8.
   If the last digit is 0 or 2, then there are $7 \cdot 8 \cdot 7$ possibilities.
   (fix the fourth digit; choose 3-9 for the first digit, have 8 choices left for the second, 7 for the third)

   If the last digit is 4, 6, or 8, there are $6 \cdot 8 \cdot 7$ possibilities
   (fix the fourth digit; for first, choose 3-9 but NOT the end digit, have 8 choices left for the second, 7 for the third).

   Total: $2(7 \cdot 8 \cdot 7) + 3(6 \cdot 8 \cdot 7)$.

2. (5 points) Every student at Accardo University is assigned a different ID number. An ID number has 6 digits, and each digit can be any of the integers 0 through 9. How many students must be in a class, in order to be assured that at least two students have IDs whose digits sum to the same number?
   (for example, the digits of 100102 and 030001 sum to the same number)

   Solution: The smallest and largest possible sums are 0 (000000) and 54 (999999), a total of 55 possible sums.

   By the Pigeonhole Principle, if we are given $55 + 1 = 56$ IDs, then at least two IDs have the same sum (56 objects, 55 boxes).

   Therefore, the answer is 56.

3. (3 pts) A collection of 5 rats, 10 mice, and 14 locusts will march in a single-file line to Bobby Dodd Stadium. How many possible lines are there, if the rats take the first five spots in the line and the mice walk together in a group?

Solution: The mice must take slots 6-15, 7-16, ..., or 20-29. Therefore, there are 15 possible configurations.

Given any configuration, the rats can be arranged in 5! ways at the front, the mice in 10! ways wherever they are, and the locusts in 14! ways in their slots.

Therefore, the number of possible lines is $15 \cdot 5! \cdot 10! \cdot 14!$.   (or $5! \cdot 10! \cdot 15!$)

Alternatively, we can view the mice as 1 entity in line and the locusts as 14 individuals, so that there are 15 entities to put in order behind the rats. (with the 1 "mouse entity" having 10! permutations of itself)

This gives us the same answer: $5! \cdot 10! \cdot 15!$.

4. (3 points) A woman has 10 friends, and she wishes to invite 5 of them over for a garden party. However, three of her friends are Aladena, Brad, and Christina, who hate other and will not attend unless the other two are not invited.

In how many ways can the woman have 5 friends over for a garden party?

Solution: Either the woman excludes the three haters (thus, chooses all 5 from the other 7), or she has exactly one of the haters at the party (thus, chooses 4 from the other 7 and 1 from the 3 haters).

Therefore, there are $\binom{7}{5} + \binom{7}{4}\binom{3}{1}$ ways that she can invite 5 friends over.