Quiz 6, Discrete Math (15 points), Fall 2016

The quiz is 20 minutes. Show your work and justify your answers where appropriate. If you write the correct answer without sufficient work or justification, you will receive little or no credit.

1. (4 points) Find the sum of the first 20 terms of the arithmetic sequence

$$a_1 = 1, \qquad a_2 = 5, \qquad a_3 = 9, \qquad a_4 = 13, \qquad \ldots.$$

2. (4 points) Solve the recurrence relation

$$a_0 = 2, \quad a_1 = 3, \qquad a_n = 6a_{n-1} - 9a_{n-2} \quad (n \geq 2).$$

3. (7 points) Use induction to prove that for every integer $n \geq 2$, we have

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} \; < \; 2 - \frac{1}{n}.$$