Quiz 5, Discrete Math (15 points), Fall 2016

The quiz is 20 minutes. Show your work and justify your answers where appropriate. If you write the correct answer without sufficient work or justification, you will receive little or no credit.

1. (1 point each) Clearly circle your answer (no justification needed here, and no partial credit given).

   (a) The digits $a_1, a_2, \ldots, a_{10}$ of an ISBN are chosen so that $a_1 + 2a_2 + 3a_3 + \cdots + 9a_9 + 10a_{10} \equiv 0 \pmod{11}$.

   [TRUE] FALSE

   (b) Suppose I attempt to copy my textbook's valid ISBN, but I accidentally type two of its digits incorrectly. Is it possible that the result is a valid ISBN? [YES] NO

   (demonstrated in the homework, 4.5 #6b)

2. (3 points) Find all integers x satisfying $0 \leq x < 30$ and $2x \equiv 4 \pmod{30}$.

   Solution: $2x \equiv 4 \pmod{30} \iff 30 \mid (2x-4) \iff 2x - 4 = 30k$ some $k \in \mathbb{N}$
   $\iff x - 2 = 15k$ some $k \in \mathbb{N} \iff x \equiv 2 \pmod{15}$.

   Thus $x = 2$ and $x = 17$ are the solutions.

3. (4 points) The number 977 is prime. Find $2^{979} \pmod{977}$.

   By Fermat's Little Theorem, since 977 is prime and 977 does not divide 2, we have

   $$2^{976} \equiv 1 \pmod{977}, \quad \text{so} \quad 2^{979} = 2^{976} \cdot 2^3 \equiv 1 \cdot 8 \pmod{977} \equiv 8 \pmod{977}.$$

   Thus, $8 = 2^{979} \pmod{977}$.

4. (6 points) Use the techniques of section 4.5 to find the smallest positive integer x satisfying

$$x \equiv 1 \pmod{7}$$

$$x \equiv 2 \pmod{50}$$

(you will receive little or no credit if you merely guess and check to try to find x)

This is a basic application of the Chinese Remainder Theorem. We have

$$x \equiv a \pmod{m}$$

$$x \equiv b \pmod{n}$$

where $\gcd(m, n) = 1$. First we must write $1 = sm + tn$ for integers s and t.

$$1 = 7s + 50t \qquad s = -7, \quad t = 1.$$

$$x = bsm + atn = 2(-7)(7) + 50 = -48.$$

However, we want x to be positive. Recall that x is unique modulo $7 \cdot 50 = 350$, so we add 350 to -48 to get

$$\boxed{x = 302}.$$