Math 1553 Worksheet: Chapter 4 and 5.1

1. Let $A = \begin{pmatrix} 7 & 1 & 4 & 1 \\ -1 & 0 & 0 & 6 \\ 9 & 0 & 2 & 3 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

a) Compute $\det(A)$.

b) Compute $\det(A^{-1})$ without doing any more work.

c) Compute $\det((A^T)^5)$ without doing any more work.

d) Find the volume of the parallelepiped formed by the columns of A.

2. Play matrix tic-tac-toe!

Instead of X against O, we have 1 against 0. The 1-player wins if the final matrix has nonzero determinant, while the 0-player wins if the determinant is zero. You can change who goes first, and you can also modify the size of the matrix.

Click the link above, or copy and paste the url below:

`http://textbooks.math.gatech.edu/ila/demos/tictactoe/tictactoe.html`

Can you think of a winning strategy for the 0 player who goes first in the 2×2 case? Is there a winning strategy for the 1 player if they go first in the 2×2 case?

3. Let A be an $n \times n$ matrix.

a) If $\det(A) = 1$ and c is a scalar, what is $\det(cA)$?

b) Using cofactor expansion, explain why $\det(A) = 0$ if A has adjacent identical columns.

4. Is there a 3×3 matrix A with only real entries, such that $A^6 = -I$? Either write such an A, or show that no such A exists.

5. In this problem, you need not explain your answers; just circle the correct one(s).

Let A be an $n \times n$ matrix.

a) Which **one** of the following statements is correct?

1. An eigenvector of A is a vector v such that $Av = \lambda v$ for a nonzero scalar λ.
2. An eigenvector of A is a nonzero vector v such that $Av = \lambda v$ for a scalar λ.
3. An eigenvector of A is a nonzero scalar λ such that $Av = \lambda v$ for some vector v.
4. An eigenvector of A is a nonzero vector v such that $Av = \lambda v$ for a nonzero scalar λ.

b) Which **one** of the following statements is **not** correct?

1. An eigenvalue of A is a scalar λ such that $A-\lambda I$ is not invertible.
2. An eigenvalue of A is a scalar λ such that $(A-\lambda I)v=0$ has a solution.
3. An eigenvalue of A is a scalar λ such that $Av=\lambda v$ for a nonzero vector v.
4. An eigenvalue of A is a scalar λ such that $\det(A-\lambda I)=0$.

6. True or false: If v_1 and v_2 are linearly independent eigenvectors of an $n\times n$ matrix A, then they must correspond to different eigenvalues.

7. In what follows, T is a linear transformation with matrix A. Find the eigenvectors and eigenvalues of A without doing any matrix calculations. (Draw a picture!)

a) $T=$ projection onto the xz-plane in $\mathbf{R}^3$.

b) $T=$ reflection over $y=2x$ in $\mathbf{R}^2$.