Math 1553 Conceptual question list §§2.6-3.6

Worksheet 5 (2.6-3.2)

1. Circle **TRUE** if the statement is always true, and circle **FALSE** otherwise.

a) If A is a 3×10 matrix with 2 pivots in its RREF, then $\dim(\text{Nul}A) = 8$ and $\text{rank}(A) = 2$.

TRUE **FALSE**

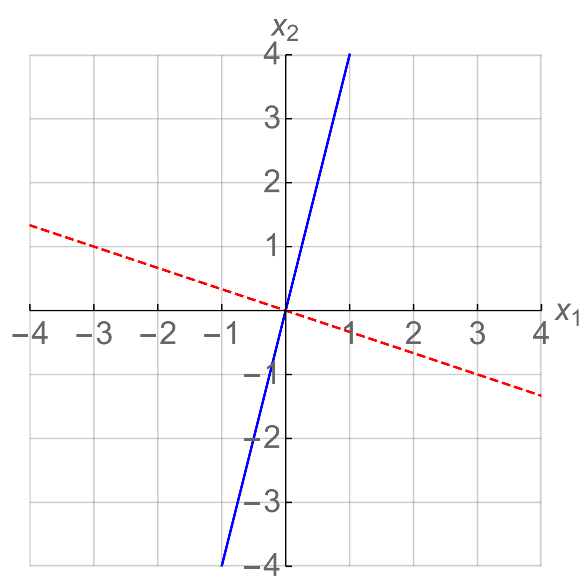
b) If A is an $m \times n$ matrix and $Ax = 0$ has only the trivial solution, then the transformation $T(x) = Ax$ is onto.

TRUE **FALSE**

c) If $\{a, b, c\}$ is a basis of a linear space V, then $\{a, a+b, b+c\}$ is a basis of V as well.

TRUE **FALSE**

2. Write a matrix A so that Col(A) is the solid blue line and Nul(A) is the dotted red line drawn below.



supplemental (2.6-3.2)

1. Circle **TRUE** if the statement is always true, and circle **FALSE** otherwise.

a) If A is a 3×100 matrix of rank 2, then $\dim(\mathrm{Nul} A) = 97$.

TRUE **FALSE**

b) If A is an $m \times n$ matrix and $Ax = 0$ has only the trivial solution, then the columns of A form a basis for $\mathbf{R}^m$.

TRUE **FALSE**

c) The set $V = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \text{ in } \mathbf{R}^4 \;\middle|\; x - 4z = 0 \right\}$ is a subspace of $\mathbf{R}^4$.

TRUE **FALSE**

2. Circle **T** if the statement is always true, and circle **F** otherwise. You do not need to explain your answer.

a) If $\{v_1, v_2, v_3, v_4\}$ is a basis for a subspace V of $\mathbf{R}^n$, then $\{v_1, v_2, v_3\}$ is a linearly independent set.

b) The solution set of a consistent matrix equation $Ax = b$ is a subspace.

c) A translate of a span is a subspace.

3. True or false (justify your answer). Answer true if the statement is *always* true. Otherwise, answer false.

a) There exists a 3×5 matrix with rank 4.

b) If A is an 9×4 matrix with a pivot in each column, then

$$\mathrm{Nul} A = \{0\}.$$

c) There exists a 4×7 matrix A such that nullity $A = 5$.

d) If $\{v_1, v_2, \ldots, v_n\}$ is a basis for $\mathbf{R}^4$, then $n = 4$.

4. **a)** True or false: If A is an $m \times n$ matrix and $\mathrm{Nul}(A) = \mathbf{R}^n$, then $\mathrm{Col}(A) = \{0\}$.

b) Give an example of 2×2 matrix whose column space is the same as its null space.

c) True or false: For some m, we can find an $m \times 10$ matrix A whose column span has dimension 4 and whose solution set for $Ax = 0$ has dimension 5.

5. Fill in the blanks: If A is a 7×6 matrix and the solution set for $Ax = 0$ is a plane, then the column space of A is a ________-dimensional subspace of $\mathbf{R}^{\square}$.

6. True or false. If the statement is *always* true, answer TRUE. Otherwise, circle FALSE.

a) The matrix transformation $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ performs reflection across the x-axis in $\mathbf{R}^2$. TRUE FALSE

b) The matrix transformation $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ performs rotation counterclockwise by 90° in $\mathbf{R}^2$. TRUE FALSE

7. Let A be a 3×4 matrix with column vectors v_1, v_2, v_3, v_4, and suppose $v_2 = 2v_1 - 3v_4$. Consider the matrix transformation $T(x) = Ax$.

a) Is it possible that T is one-to-one? If yes, justify why. If no, find distinct vectors v and w so that $T(v) = T(w)$.

b) Is it possible that T is onto? Justify your answer.

8. Answer each question.

a) Suppose $S : \mathbf{R}^3 \to \mathbf{R}^2$ is the matrix transformation $S(x) = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{pmatrix} x$.

Is S one-to-one? YES NO

Is S onto? YES NO

b) Suppose $T : \mathbf{R}^2 \to \mathbf{R}^2$ is given by $T(x, y) = (x - y,\ x - y)$.
Is T one-to-one? YES NO

Is T onto? YES NO

c) Suppose $T : \mathbf{R}^n \to \mathbf{R}^m$ is a one-to-one matrix transformation. Which one of the following *must* be true? (cicle one)
$m = n$ $m < n$ $m \leq n$ $m > n$ $m \geq n$

9. Which of the following transformations are onto?
Circle all that apply.

a) $T : \mathbf{R}^2 \to \mathbf{R}^2$ that rotates counterclockwise by $\dfrac{\pi}{12}$ radians.

b) The transformation $T(x) = Ax$, where A is a 4×3 matrix with three pivots.

c) $T : \mathbf{R}^3 \to \mathbf{R}^3$ that reflects across the yz-plane.

Worksheet 6 (3.3-3.4)

1. If A is a 3×5 matrix and B is a 3×2 matrix, which of the following are defined?

a) $A - B$

b) AB

c) $A^T B$

d) $B^T A$

e) A^2

2. A is $m \times n$ matrix, B is $n \times m$ matrix. Select proper answers from the box. Multiple answers are possible

a) Take any vector x in $\mathbf{R}^m$, then ABx *must be* in:

| Col(A), Nul(A), Col(B), Nul(B) |

b) Take any vector x in $\mathbf{R}^n$, then BAx *must be* in:

| Col(A), Nul(A), Col(B), Nul(B) |

c) If $m > n$, then columns of AB could be linearly | *independent*, *dependent* |

d) If $m > n$, then columns of BA could be linearly | *independent*, *dependent* |

e) If $m > n$ and $Ax = 0$ has nontrivial solutions, then columns of BA could be linearly | *independent*, *dependent* |

Supplemental (3.3-3.4)

1. Circle **T** if the statement is always true, and circle **F** otherwise.

a) **T** **F** If $T : \mathbf{R}^n \to \mathbf{R}^n$ is linear and $T(e_1) = T(e_2)$, then the homogeneous equation $T(x) = 0$ has infinitely many solutions.

b) **T** **F** If $T : \mathbf{R}^n \to \mathbf{R}^m$ is a one-to-one linear transformation and $m \neq n$, then T must not be onto.

2. Consider $T : \mathbf{R}^3 \to \mathbf{R}^3$ given by

$$T(x, y, z) = (x,\ x + z,\ 3x - 4y + z,\ x).$$

Is T one-to-one? Justify your answer.

3. In each case, determine whether T is linear. Briefly justify.

a) $T(x_1, x_2) = (x_1 - x_2, x_1 + x_2, 1)$.

b) $T(x, y) = (y, x^{1/3})$.

c) $T(x, y, z) = 2x - 5z$.

4. True or false (justify your answer). Answer true if the statement is *always* true. Otherwise, answer false.

a) If A and B are matrices and the products AB and BA are both defined, then A and B must be square matrices with the same number of rows and columns.

b) If A, B, and C are nonzero 2×2 matrices satisfying $BA = CA$, then $B = C$.

c) Suppose A is an 4×3 matrix whose associated transformation $T(x) = Ax$ is not one-to-one. Then there must be a 3×3 matrix B which is not the zero matrix and satisfies $AB = 0$.

d) Suppose $T : \mathbf{R}^n \to \mathbf{R}^m$ and $U : \mathbf{R}^m \to \mathbf{R}^p$ are one-to-one linear transformations. Then $U \circ T$ is one-to-one. (What if U and T are not necessarily linear?)

5. In each case, use geometric intuition to either give an example of a matrix with the desired properties or explain why no such matrix exists.

a) A 3×3 matrix P, which is not the identity matrix or the zero matrix, and satisfies $P^2 = P$.

b) A 2×2 matrix A satisfying $A^2 = I$.

c) A 2×2 matrix A satisfying $A^3 = -I$.

Worksheet 7 (3.5-3.6)

1. True or false (justify your answer). Answer true if the statement is *always* true. Otherwise, answer false.

a) If A and B are $n \times n$ matrices and both are invertible, then the inverse of AB is $A^{-1}B^{-1}$.

b) If A is an $n \times n$ matrix and the equation $Ax = b$ has at least one solution for each b in $\mathbf{R}^n$, then the solution is *unique* for each b in $\mathbf{R}^n$.

c) If A is an $n \times n$ matrix and the equation $Ax = b$ has at most one solution for each b in $\mathbf{R}^n$, then the solution must be *unique* for each b in $\mathbf{R}^n$.

d) If A and B are invertible $n \times n$ matrices, then $A+B$ is invertible and $(A+B)^{-1} = A^{-1} + B^{-1}$.

e) If A and B are $n \times n$ matrices and $ABx = 0$ has a unique solution, then $Ax = 0$ has a unique solution.

f) If A is a 3×4 matrix and B is a 4×2 matrix, then the linear transformation Z defined by $Z(x) = ABx$ has domain $\mathbf{R}^3$ and codomain $\mathbf{R}^2$.

g) Suppose A is an $n \times n$ matrix and every vector in $\mathbf{R}^n$ can be written as a linear combination of the columns of A. Then A must be invertible.