Supplemental problems: §§2.6, 2.7, 2.9

1. Circle **TRUE** if the statement is always true, and circle **FALSE** otherwise.

a) If A is a 3×100 matrix of rank 2, then $\dim(\mathrm{Nul}A) = 97$.

TRUE **FALSE**

b) If A is an $m \times n$ matrix and $Ax = 0$ has only the trivial solution, then the columns of A form a basis for $\mathbf{R}^m$.

TRUE **FALSE**

c) The set $V = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \text{ in } \mathbf{R}^4 \;\middle|\; x - 4z = 0 \right\}$ is a subspace of $\mathbf{R}^4$.

TRUE **FALSE**

2. Write a matrix A so that $\mathrm{Col}A = \mathrm{Span}\left\{ \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} \right\}$ and $\mathrm{Nul}A$ is the xz-plane.

3. Circle **T** if the statement is always true, and circle **F** otherwise. You do not need to explain your answer.

a) If $\{v_1, v_2, v_3, v_4\}$ is a basis for a subspace V of $\mathbf{R}^n$, then $\{v_1, v_2, v_3\}$ is a linearly independent set.

b) The solution set of a consistent matrix equation $Ax = b$ is a subspace.

c) A translate of a span is a subspace.

4. True or false (justify your answer). Answer true if the statement is *always* true. Otherwise, answer false.

a) There exists a 3×5 matrix with rank 4.

b) If A is an 9×4 matrix with a pivot in each column, then

$$\mathrm{Nul}A = \{0\}.$$

c) There exists a 4×7 matrix A such that nullity $A = 5$.

d) If $\{v_1, v_2, \ldots, v_n\}$ is a basis for $\mathbf{R}^4$, then $n = 4$.

5. Find bases for the column space and the null space of

$$A = \begin{pmatrix} 0 & 1 & -3 & 1 & 0 \\ 1 & -1 & 8 & -7 & 1 \\ -1 & -2 & 1 & 4 & -1 \end{pmatrix}.$$

6. Find a basis for the subspace V of $\mathbf{R}^4$ given by

$$V = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \text{ in } \mathbf{R}^4 \;\middle|\; x + 2y - 3z + w = 0 \right\}.$$

7. **a)** True or false: If A is an $m \times n$ matrix and $\text{Nul}(A) = \mathbf{R}^n$, then $\text{Col}(A) = \{0\}$.

b) Give an example of 2×2 matrix whose column space is the same as its null space.

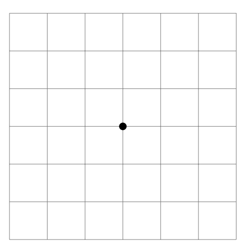
c) True or false: For some m, we can find an $m \times 10$ matrix A whose column span has dimension 4 and whose solution set for $Ax = 0$ has dimension 5.

8. Suppose V is a 3-dimensional subspace of $\mathbf{R}^5$ containing $\begin{pmatrix} 1 \\ -4 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -3 \\ 1 \\ 0 \end{pmatrix}$, and $\begin{pmatrix} 9 \\ 8 \\ 1 \\ 0 \\ 1 \end{pmatrix}$.

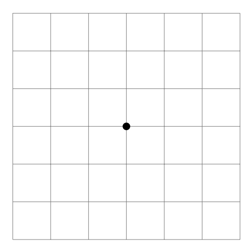
Is $\left\{ \begin{pmatrix} 1 \\ -4 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -3 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 9 \\ 8 \\ 1 \\ 0 \\ 1 \end{pmatrix} \right\}$ a basis for V? Justify your answer.

9. **a)** Write a 2×2 matrix A with **rank** 2, and draw pictures of $\text{Nul}A$ and $\text{Col}A$.

$A = \begin{pmatrix} \quad & \quad \\ \quad & \quad \end{pmatrix}$ Nul A =

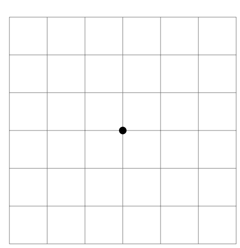


Col A =

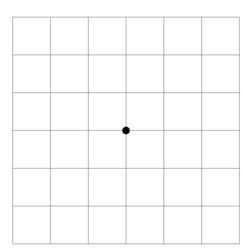


b) Write a 2×2 matrix B with **rank** 1, and draw pictures of $\text{Nul}B$ and $\text{Col}B$.

$B = \begin{pmatrix} \quad & \quad \\ \quad & \quad \end{pmatrix}$ Nul B =



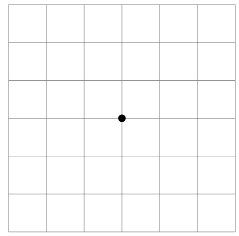
Col B =



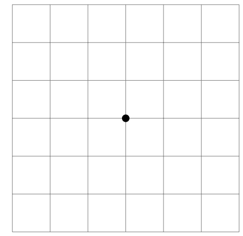
c) Write a 2×2 matrix C with **rank** 0, and draw pictures of $\operatorname{Nul} C$ and $\operatorname{Col} C$.

$$C = \begin{pmatrix} & \\ & \end{pmatrix}$$

Nul C =



Col C =



(In the grids, the dot is the origin.)