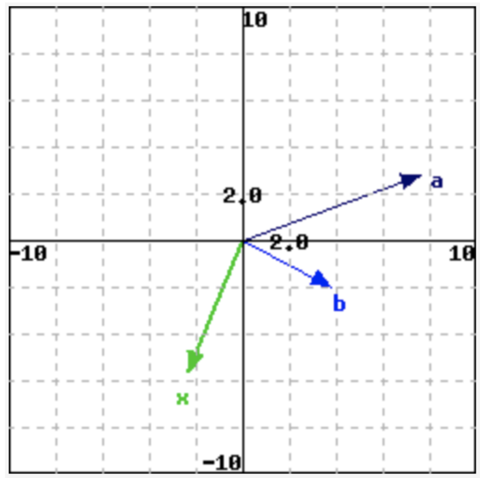
Supplemental problems: §2.1, §2.2, §2.3

1. Consider the following vectors x, a, and b.



Can we write x as a linear combination of a and b?

a) Formulate this question as a vector equation.

b) Formulate this question as a matrix equation.

c) Give an approximate answer to the question using geometric intuition rather than attempting to solve a system of equations.

2. Consider the augmented matrix

$$\left(\begin{array}{ccc|c} 2 & -2 & 2 & 0 \\ 1 & -3 & -4 & -9 \\ 3 & -1 & 8 & 9 \end{array}\right)$$

Question: Does the corresponding linear system have a solution? If so, what is the solution set?

a) Formulate this question as a vector equation.

b) Formulate this question as a system of linear equations.

c) Formulate this question as a matrix equation.

d) What does this mean in terms of spans?

e) Answer the question using the interactive demo.

f) Answer the question using row reduction.

g) Find a **different** solution in parts (e) and (d).

3. Let $v_1 = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \qquad v_2 = \begin{pmatrix} -2 \\ -3 \\ -1 \end{pmatrix} \qquad w = \begin{pmatrix} 2 \\ -4 \\ 8 \end{pmatrix}.$

Question: Is w a linear combination of v_1 and v_2? In other words, is w in $\text{Span}\{v_1, v_2\}$?

a) Formulate this question as a vector equation.

b) Formulate this question as a system of linear equations.

c) Formulate this question as an augmented matrix.

d) Formulate this question as a matrix equation.

e) Answer the question using the interactive demo.

f) Answer the question using row reduction.

4. Let

$$A = \begin{pmatrix} 1 & 0 & 5 \\ -2 & 1 & -6 \\ 0 & 2 & 8 \end{pmatrix}, \qquad b = \begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix}$$

Is b in the span of the columns of A? In other words, is b a linear combination of the columns of A? Justify your answer.

5. Consider the vector equation

$$x \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + y \begin{pmatrix} -2 \\ -1 \\ -1 \end{pmatrix} + z \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} -5 \\ -1 \\ -2 \end{pmatrix}.$$

Question: Is there a solution? If so, what is the solution set?

a) Formulate this question as an augmented matrix.

b) Formulate this question as a system of linear equations.

c) Formulate this question as a matrix equation.

d) What does this mean in terms of spans?

e) Answer the question using the interactive demo.

f) Answer the question using row reduction.

6. Let $v_1 = \begin{pmatrix} 1 \\ k \end{pmatrix}$, $v_2 = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$, and $b = \begin{pmatrix} 1 \\ h \end{pmatrix}$.

a) Find all values of h and k so that $x_1 v_1 + x_2 v_2 = b$ has infinitely many solutions.

b) Find all values of h and k so that b is *not* in $\text{Span}\{v_1, v_2\}$.

c) Find all values of h and k so that there is exactly one way to express b as a linear combination of v_1 and v_2.

7. Decide if each of the following statements is true or false. If it is true, prove it; if it is false, provide a counterexample.

a) Every set of four or more vectors in $\mathbf{R}^3$ will span $\mathbf{R}^3$.

b) The span of any set contains the zero vector.

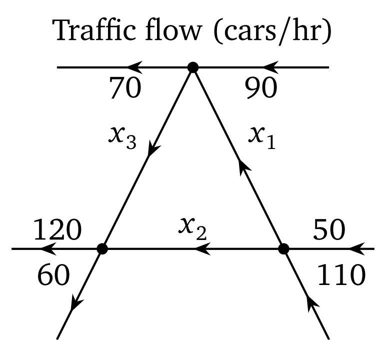
8. Is $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ in the span of $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$? Justify your answer.

9. Acme Widgets, Gizmos, and Doodads has two factories. Factory A makes 10 widgets, 3 gizmos, and 2 doodads every hour, and factory B makes 4 widgets, 1 gizmo, and 1 doodad every hour.

a) If factory A runs for a hours and factory B runs for b hours, how many widgets, gizmos, and doodads are produced? Express your answer as a vector equation.

b) A customer places an order for 16 widgets, 5 gizmos, and 3 doodads. Can Acme fill the order with no widgets, gizmos, or doodads left over? If so, how many hours do the factories run? If not, why not?

10. The diagram below represents traffic in a city.



a) Write a system of three linear equations whose solution would give the values of x_1, x_2, and x_3. Do not solve it.

b) Write the system of equations as a vector equation. Do not solve it.

11. True or false. If the statement is *ever* false, answer false. Justify your answer.

a) A matrix equation $Ax = b$ is consistent if A has a pivot in every column.

b) If an $m \times n$ matrix A has fewer than n pivots and b is in $\mathbf{R}^m$, then $Ax = b$ has infinitely many solutions.

c) Suppose A is a 3×3 matrix and there is a vector y in $\mathbf{R}^3$ so that $Ax = y$ does not have a solution. Is it possible that there is a z in $\mathbf{R}^3$ so that the equation $Ax = z$ has a *unique* solution? Justify your answer.

d) There is a matrix A and a nonzero vector b so that the solution set of $Ax = b$ is a plane through the origin.

e) Suppose A is an $m \times n$ matrix and b is in $\mathbf{R}^m$. If the columns of A span $\mathbf{R}^m$, then $Ax = b$ must be consistent.