Math 1553 Worksheet §§5.1, 5.2, 5.4

1. Answer yes, no, or maybe. Justify your answers. In each case, A is a matrix whose entries are real numbers.

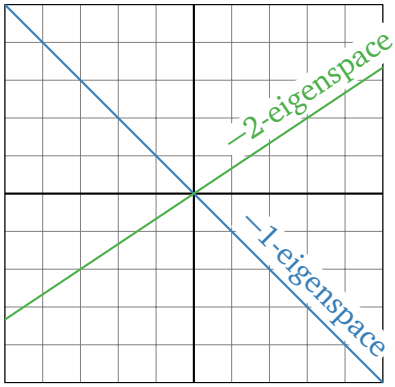
a) Suppose $A = \begin{pmatrix} 3 & 0 & 0 \\ 5 & 1 & 0 \\ -10 & 4 & 7 \end{pmatrix}$. Then the characteristic polynomial of A is

$$\det(A - \lambda I) = (3-\lambda)(1-\lambda)(7-\lambda).$$

b) If A is a 3×3 matrix with characteristic polynomial $-\lambda(\lambda-5)^2$, then the 5-eigenspace is 2-dimensional.

c) If A is an invertible 2×2 matrix, then A is diagonalizable.

2. The eigenspaces of some 2×2 matrix A are drawn below. Write an invertible matrix C and a diagonal matrix D so that $A = CDC^{-1}$.



3. Let

$$A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^{-1}.$$

Find a formula for A^n (where n is a positive integer).