**Math 1553 Quiz 6, Spring 2020**

Solutions

**1.** This was the honor code statement.

**2.** We are told $\det(A) = -1$ and asked to find $\det(A^{-1})$.

$$\det(A^{-1}) = \frac{1}{\det(A)} = \frac{1}{-1} = -1.$$

**3.** We need the area of the triangle in $\mathbf{R}^2$ with vertices $(1,2)$, $(4,3)$, and $(2,5)$. The vector from the first vertex to the second is $v_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, and the vector from the first vertex to the third is $v_2 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$. The triangle has half the area of the parallelogram naturally determined by $v_1$ and $v_2$, so

$$\text{Area of triangle} = \frac{1}{2}\left|\det\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}\right| = \frac{1}{2}(8) = 4.$$

**4.** To find $\det\begin{pmatrix} 1 & 2 & 0 \\ 2 & 3 & 0 \\ 5 & 4 & 1 \end{pmatrix}$, we can use the cofactor expansion along the third column.

$$\det\begin{pmatrix} 1 & 2 & 0 \\ 2 & 3 & 0 \\ 5 & 4 & 1 \end{pmatrix} = (-1)^{3+3}(1)\det\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} = (1)(1)(-1) = -1.$$

**5.** If $A$ is a $3 \times 3$ matrix with the first row the same as the second row, then the RREF of $A$ will have a row of zeros. Therefore, $A$ must not be invertible, which means that 0 must be an eigenvalue of $A$.

**6.** We need to find the value of $m$ so that $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is an eigenvector of $A = \begin{pmatrix} 1 & m \\ 2 & 3 \end{pmatrix}$. Note

$$A\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & m \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1+m \\ 5 \end{pmatrix}.$$

From the second entry we see that if $A\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \lambda\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ then $\lambda = 5$, in which case $A\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \end{pmatrix} = \begin{pmatrix} 1+m \\ 5 \end{pmatrix}$, so $m = 4$. We can check to verify that indeed

$$\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}.$$