Name:________________________________ Studio Section:__________

Math 1553 Quiz 5, Spring 2020 (10 points, 10 minutes)
Jankowski, Lecture C1-C4 (11:15 AM)

Solutions

You do not need to show your work except in problem 2(a) and problem 3.

1. (2 points) Suppose A is an $m \times n$ matrix with $m < n$, and let T be its associated matrix transformation $T(x) = Ax$.

a) Which of the following is correct?
(i) T cannot be one-to-one.

b) Which of the following is correct?
(ii) There is not enough information to tell if T is onto.

2. (5 points) Consider the linear transformation $T : \mathbf{R}^2 \to \mathbf{R}^3$ given by

$$T(x,y) = (2x - y,\ y - x,\ x).$$

a) Find the standard matrix A for T.

$$A = \begin{pmatrix} T(e_1) & T(e_2) \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ -1 & 1 \\ 1 & 0 \end{pmatrix}.$$

b) Is T onto? YES $\boxed{\text{NO}}$ T is a linear transformation from $\mathbf{R}^2$ to $\mathbf{R}^3$. Just from the fact that $2 < 3$ we see T cannot be onto, no work required.

c) Is T one-to-one? $\boxed{\text{YES}}$ NO Note A has two pivots, or alternatively, note that if $T(x,y) = (0,0,0)$ then from its third and second entries we get $x = 0$ and also $y - x = 0$ thus $y = 0$. Thus if $T(v) = 0$ then $v = (0,0)$.

3. (3 points) Suppose $T : \mathbf{R}^2 \to \mathbf{R}^2$ is a linear transformation satisfying

$$T\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}2\\-1\end{pmatrix} \quad \text{and} \quad T\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\3\end{pmatrix}.$$

Find $T\begin{pmatrix}2\\-1\end{pmatrix}$.

By linearity,

$$T\begin{pmatrix}2\\-1\end{pmatrix} = T\begin{pmatrix}2\\0\end{pmatrix} + T\begin{pmatrix}0\\-1\end{pmatrix} = 2T\begin{pmatrix}1\\0\end{pmatrix} - 1T\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}4\\-2\end{pmatrix} - \begin{pmatrix}1\\3\end{pmatrix} = \begin{pmatrix}3\\-5\end{pmatrix}.$$