Name:______________________________ Studio Section:__________

Math 1553 Quiz 5, Spring 2020 (10 points, 10 minutes)
Jankowski, Lecture A1-A3 (8:00 AM)

Solutions

You do not need to show your work except in problem 2(a) and problem 3.

1. (2 points) Suppose A is an $m \times n$ matrix with $m > n$, and let T be its associated matrix transformation $T(x) = Ax$.

a) Which of the following is correct?
(ii) There is not enough information to tell if T is one-to-one.

b) Which of the following is correct?
(i) T cannot be onto.

2. (5 points) Consider the linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^2$ given by

$$T(x,y,z) = (x - y - 2z,\ 2x - 2y - 4z).$$

a) Find the standard matrix A for T.

$$A = \begin{pmatrix} T(e_1) & T(e_2) & T(e_3) \end{pmatrix} = \begin{pmatrix} 1 & -1 & -2 \\ 2 & -2 & -4 \end{pmatrix}.$$

b) Is T onto? YES $\boxed{\text{NO}}$ A only has one pivot, or alternatively, the second entry in $T(v)$ is always 2 times the first, so for example $(3,1)$ is not in the range of T.

c) Is T one-to-one? YES $\boxed{\text{NO}}$ T is a linear transformation from $\mathbf{R}^3$ to $\mathbf{R}^2$. Just from the fact that $3 > 2$ we see T cannot be one-to-one, no work required.

3. (3 points) Suppose $T : \mathbf{R}^2 \to \mathbf{R}^2$ is a linear transformation satisfying

$$T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \quad \text{and} \quad T\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}.$$

Find $T\begin{pmatrix} 2 \\ 4 \end{pmatrix}$.

By linearity,

$$T\begin{pmatrix} 2 \\ 4 \end{pmatrix} = T\begin{pmatrix} 2 \\ 0 \end{pmatrix} + T\begin{pmatrix} 0 \\ 4 \end{pmatrix} = 2T\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 4T\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \end{pmatrix} + \begin{pmatrix} -4 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}.$$