

Math 1553 Worksheet §4.4 and 4.5, Matrix Multiplication and Inverses

1. True or false (justify your answer). Answer true if the statement is *always* true. Otherwise, answer false.
- a) If A is a 3×4 matrix and B is a 4×2 matrix, then the linear transformation Z defined by $Z(x) = ABx$ has domain \mathbf{R}^2 and codomain \mathbf{R}^3 .
- b) If A is an $n \times n$ matrix and the equation $Ax = b$ has at least one solution for each b in \mathbf{R}^n , then the solution is *unique* for each b in \mathbf{R}^n .
- c) Suppose A is an $n \times n$ matrix and every vector in \mathbf{R}^n can be written as a linear combination of the columns of A . Then A must be invertible.
- d) Suppose $T : \mathbf{R}^n \rightarrow \mathbf{R}^m$ and $U : \mathbf{R}^m \rightarrow \mathbf{R}^p$ are linear transformations and $U \circ T$ is onto. Then U and T must both be onto.

2. Let $T : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ be rotation *clockwise* by 60° . Let $U : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ be the linear transformation with standard matrix $\begin{pmatrix} -2 & 1 \\ 1 & 0 \end{pmatrix}$.

a) Find the standard matrix for the composition $U \circ T$.

b) Find the standard matrix for the composition $T \circ U$.

c) Find the standard matrix for U^{-1} .