Math 1553 Worksheet §3.6, 3.7, 3.9, 4.1

1. Circle **TRUE** if the statement is always true, and circle **FALSE** otherwise.

a) If A is a 3×100 matrix of rank 2, then $\dim(\text{Nul}A) = 97$.

TRUE **FALSE**

b) If A is an $m \times n$ matrix and $Ax = 0$ has only the trivial solution, then the columns of A form a basis for $\mathbf{R}^m$.

TRUE **FALSE**

c) The set $V = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \text{ in } \mathbf{R}^4 \;\middle|\; x - 4z = 0 \right\}$ is a subspace of $\mathbf{R}^4$.

TRUE **FALSE**

2. Write a matrix A so that $\text{Col}A = \text{Span}\left\{ \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} \right\}$ and $\text{Nul}A$ is the xz-plane.

3. Let $A = \begin{pmatrix} 1 & -5 & -2 & -4 \\ 2 & 3 & 9 & 5 \\ 1 & 1 & 4 & 2 \end{pmatrix}$, and let T be the matrix transformation associated to A, so $T(x) = Ax$.

a) What is the domain of T? What is the codomain of T? Give an example of a vector in the range of T.

b) The RREF of A is $\begin{pmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$. Is there a vector in the codomain of T which is not in the range of T? Justify your answer.