

Name: _____

Recitation Section: _____

Math 1553 Quiz 5, Spring 2019 (10 points, 10 minutes)**Solutions**

1. Answer each question. No work is necessary for this problem.

a) Suppose $S : \mathbf{R}^3 \rightarrow \mathbf{R}^2$ is the matrix transformation $S(x) = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{pmatrix}x$.

Is S one-to-one? NO

Is S onto? YES

b) Suppose $T : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ is given by $T(x, y) = (x - y, x - y)$.

Is T one-to-one? NO

Is T onto? NO

c) Suppose $T : \mathbf{R}^n \rightarrow \mathbf{R}^m$ is a one-to-one matrix transformation. Which one of the following *must* be true? (circle one)

$$m \geq n$$

2. (5 points) Let $T : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ be the linear transformation that first reflects across the x -axis, then rotates clockwise by 45° . Find the standard matrix A for T .

Show your steps clearly and simplify all values of trigonometric functions (do not leave your answer in terms of sines and cosines).

Solution: $A = (T(e_1) \ T(e_2))$. Here, e_1 is fixed by reflection across the x -axis, then rotated 45° clockwise to arrive at $(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$. Also, e_2 is flipped to $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ then rotated clockwise 45° to arrive at $(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$.

$$e_1 : \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightsquigarrow \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}, \quad e_2 : \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 0 \\ -1 \end{pmatrix} \rightsquigarrow \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}.$$

So

$$A = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}.$$

It's also possible to do the problem with matrix multiplication if you wish, since we did it in class so soon after 4.2 and 4.3.

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}.$$