Math 1553 Supplement: Chapter 5 and §6.1

1. Find the volume of the parallelepiped in $\mathbf{R}^4$ naturally determined by the vectors

$$\begin{pmatrix} 4 \\ 1 \\ 3 \\ 8 \end{pmatrix}, \begin{pmatrix} 0 \\ 7 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 5 \\ -5 \\ 0 \\ 7 \end{pmatrix}.$$

2. If A is a 3×3 matrix and $\det(A) = 1$, what is $\det(-2A)$?

3. **a)** Is there a real 2×2 matrix A that satisfies $A^4 = -I_2$? Either write such an A, or show that no such A exists.
(hint: think geometrically! The matrix $-I_2$ represents rotation by π radians).

b) Is there a real 3×3 matrix A that satisfies $A^4 = -I_3$? Either write such an A, or show that no such A exists.

4. Match the statements (i)-(v) with the corresponding statements (a)-(e). All matrices are 3×3. There is a unique correspondence. Justify the correspondences in words.

(i) $Ax = \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix}$ has a unique solution.
(ii) The transformation $T(v) = Av$ fixes a nonzero vector.
(iii) A is obtained from B by subtracting the third row of B from the first row of B.
(iv) The columns of A and B are the same; except that the first, second and third columns of A are respectively the first, third, and second columns of B.
(v) The columns of A, when added, give the zero vector.

(a) 0 is an eigenvalue of A.
(b) A is invertible.
(c) $\det(A) = \det(B)$
(d) $\det(A) = -\det(B)$
(e) 1 is an eigenvalue of A.

5. True or false. If the statement is always true, answer true and justify why it is true. Otherwise, answer false and give an example that shows it is false. In every case, assume that A is an $n \times n$ matrix.

a) The diagonal entries of A are its eigenvalues.

b) If A is invertible and 2 is an eigenvalue of A, then $\frac{1}{2}$ is an eigenvalue of A^{-1}.

6. Find a basis $\mathcal{B}$ for the (-1)-eigenspace of $Z = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 2 & 4 \\ 0 & 0 & -1 \end{pmatrix}$

7. Suppose A is an $n \times n$ matrix satisfying $A^2 = 0$. Find all eigenvalues of A. Justify your answer.